LAND MARKETS AND PUBLIC POLICY

AN ACT OF BALANCE IN SPATIAL EQUILIBRIUM

PROEFSCHRIFT

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‘De stad die het ergste van woonverval had toegelaten was de eerste geweest om zich een nieuw gebod op te leggen, en daarmee de wereld een elfde: Gij - zult - ruim - wonen.’
Ferdinand Bordewijk, *Keizerrijk*

Chapter 1

Introduction

1.1 Land markets and public policy

The organisation of human social and economic life is constrained by time as well as space. The spatial component does not only pose limitations in the form of distances, but can also offer possibilities for development as a result of favourable location characteristics. Some of the characteristics of a location are dictated by natural circumstances such as climate, altitude, and the presence of water and natural resources. Others result from human activity: agriculture, cities, roads, and ports. There often exists a clear dependency between the two parts, though sometimes human activity seems to overrule nature. With sufficient cultivation, agriculture can be situated in areas that do not have a favourable soil; some major cities are vulnerable to earthquakes or lie below sea level.

A relatively recent concern in many regions of the world is the protection of nature against any form of human use. This can apply to the conservation of specific animal or plant species to avoid extinction, or to the protection of entire habitats and even ecosystems. Indirect benefits from protection are often related to climate change issues and deforestation. Forests can secure CO_2 absorption. Refraining from certain types of agriculture can reduce the emission of CO_2 and other green house gasses such as methane. Besides these indirect benefits, the presence of nature is generally also considered to contribute to an overall ‘quality of life’. Many people like the view of a landscape that is not dominated by housing or industry. And even if the view itself is not enjoyed frequently, there exists—at least in countries and regions with a high population density—a concern about the conservation of *open space*, as land that is simply not used as a built environment. In these areas, where undeveloped land has become a scarce resource mainly for its quality as open space, governments impose restrictions upon development by means of zoning. Instead of prohibiting people to enter certain areas, governments might introduce national parks as a means of protecting areas against resource exploitation, while simultaneously allowing for recreational and educational use. This combination can be applied especially for the protection of open space, because part of recreational use might be enjoying wide views, even if they concern agricultural land rather than a natural reserve. Although the amount of protected areas and the degree of protection may vary across countries and continents, particularly in smaller densely populated regions (Western Europe and especially the Netherlands), finding the right balance between land use and protection has become a major concern in public policy (Centraal Planbureau, 1999; Ministerie van VROM, 2000, 2001). National governments can respond with *land use planning*, sometimes combined with regional development programmes. Landscape planning might be an issue at a regional level, with the image of the landscape being protected under the UNESCO Heritage programme. Some regions define their competitive advantages not only in terms of economic and social structure but also

Figure 1.1: *Open space as an excavation near Enschede (NL)*

by the presence of natural amenities, in order to attract economic activity or to keep it within their borders. At the local level, municipalities often take care of the legislation for—or in the Netherlands frequently even the supply of—residential housing, industrial sites, recreational parks, and green belts. By incorporating conditions in contracts when selling land to third parties, local governments acting as private owners can implement environmental policy goals for specific sites or industries. In most countries a substantial part of the total area of land is privately owned. The ownership of a parcel might change in these countries by trading the land on a private market. As a consequence, public policy with respect to land use—often labelled *land policy*—also needs to address the *land market*. Marking the position of a government relative to a market is an important topic in *economics of the public sector*. The traditional approaches in this sub-discipline of economics rely heavily on the neoclassical general equilibrium framework and its *Welfare Theorems*. These theorems suggest that under specified conditions an allocation of goods by a market is to be preferred over a central allocation by the state. Exceptions apply to goods that do not allow for the isolation of individual consumption, the so-called *public goods* and goods that induce *external effects*. Whether the Welfare Theorems can be applied to the allocation of land and whether they can serve as a basis for land policy are the main questions to be addressed in this thesis.

Recent literature in spatial economics follows earlier observations by economic theorists, showing that the neoclassical framework is unsuitable for explaining the

Figure 1.2: *Environmentalists' action concerning the conservation of open space near Leiden (NL)*

formation of agglomerations. Because a theory of agglomeration formation is indispensable in a welfare assessment of land policy, it is likely to be necessary to first redefine the concept of welfare—or *well-being*—in a spatial context. Once the concept of *spatial welfare* is clarified, it might serve as the basis in the assessment of *sustainable* land use planning. The concept of spatial welfare can therefore be considered as a first step in the *transition* toward sustainable land policies.

1.2 Spatial welfare

Many changes of land use types are irreversible, at least for some years or decades. In addition, locations themselves are always geographically unique, and are rarely isolated. The concern about an *optimal*—or at least 'right', however that is defined—land use type is closely tied to the *value* that people can assign to a particular parcel. This value does not need to be expressed in monetary terms directly. Following microeconomic theory, a market price is rather the monetary valuation of the trade-off people experience in consuming more of one good at the expense of another. If the trade-off is derived from some measure designating the level of *well-being*, the meaning of the word value is related to this level of well-being first and the market prices second. In that case, instead of 'value' the term *social value* is occasionally used in economic literature. The optimality of a market allocation then depends on the correspondence between social value and market price.

Adopting this usage of the term value, the social value of land can be thought of as a combination of two other values:

1. The value of the land itself,
2. The value of the land use type, or the good on the land.

In the case of commercial use of land, the market price often refers to the second value, while it is actually combined with the first. The price of a new house, for example, is usually higher than the total costs of building it. Price minus the building costs represent the value of the land on which the house is built. Whether the market price of land corresponds exactly to its social value depends on how well the land market operates in economic theoretic terms. This question (the main research question in this study) will be addressed in more detail in section 1.3. The social value of protected areas is usually not made explicit by a market price, perhaps with the exception of privately owned parks for which an entrance fee is charged for visitors. In most cases the value of the protected area concerns the second value (the land use type) for example the ecosystem it is part of. Sometimes, the value of a protected area of land might also be expressed as the potential market price the land would yield in case of development, determined—in turn—by the market value of the land use type, for example residential housing.

In commercial and non-commercial use, or even ‘non-use’—for example in the case of *open space*—, the value of land can be derived in principle from a *demand* for land. The demand for land with a commercial land use type can be translated directly to location choices of consumers and producers. If—in a first assessment—it is assumed that all existing houses, offices, plants, etc. are in use in the sense that demand equals supply and no vacancy exists, the corresponding land use type is fully determined by the demand that follows the location choices of all consumers and producers. In reality, however, the situation is often far more complex. Houses can be vacant in one district of a town, while there is a shortage in another. Some regions still have plants and other facilities within their borders that belong to a type of industry that has moved to another region; often another continent. Finally, some natural areas are under constant pressure of expanding cities, where the support for environmental protection groups reveals a demand for nature instead of housing in at least some parts of society. Demand for and supply of land and the accompanying land use type do not match in many situations. Before discussing the division of roles between public policy and the land market in equating demand and supply with respect to both market and social value, more specific characteristics of land as a good in economic theory will be discussed.

1.2.1 Space as an economic good

People are usually not indifferent when choosing locations, for example if they consider buying a house. As a result, there exists some degree of monopoly for owners selling or hiring out land. In line with the discussions above, the monopoly often rather applies to the specific characteristics of the land, such as the type of house built on it. Extending the example of buying a house, although two houses can be very different, people usually buy only one. Different houses are similar goods, but individual consumers have a preference for certain characteristics of the house or its location. Therefore, housing can be considered a *differentiated* good. The land market—and thereby the housing market—could therefore be best defined as an *oligopolistic* market. Competition exists, but land owners are likely to enforce some degree of market power, depending on the demand for relatively scarce characteristics.

Development of land could be considered as the production of, for example, residential space. If a city expands, it does so usually by building new districts and thereby increasing its total surface area. Only in larger cities is the supply of space significantly increased in the centre by means of high-rise buildings. Extending the borders of residential and industrial areas comes at the loss of undeveloped land as nature or open space. Societal concern about the preservation of undeveloped land can also be interpreted as the demand for a *non-market good*. This demand can in turn be translated to a value or a contribution to social welfare or level of well-being. Tools for assessing these values have been developed for—or are at least predominantly applied in—environmental economics, such as *hedonic pricing* and the *contingent valuation method*. Although the latter can be adopted for estimating the existence value of, for example, a nature reserve, it does not directly assign a value per square meter to the land itself that can be compared to market price. Alternatively, undeveloped land could be considered the main production factor—as a natural resource—in the production of developed land, but this perspective would not do justice to the social value of nature. Even if the scarcity of land was addressed similarly to other natural resources such as oil or coal, a forest also contributes to welfare by its current presence, not only by its future use as an input factor.

These examples indicate that the value of land is a relatively difficult concept to deal with in economics. On one hand it is a differentiated good, enforcing questions with respect to market power on the side of land owners, while on the other the valuation of undeveloped land—as a non-market good—cannot readily assess the mutual exclusiveness in the choice between use and non-use for the same parcel of land. The issue of comparing the use and non-use value of land is furthermore complicated by the fact that its market price not only reflects the use value of the quantity of land, but to some degree also the value of characteristics—or the *quality*—of the parcel.

1.2.2 Agglomerations and external effects

Groups of people living in the same area might induce considerable social and economic impacts at a local level. These impacts are again part of the environment of other people. This aspect of land use highlights the need for great care in defining land use policy options from yet another perspective. In economic theoretical terms, the direct impact people have on others can be characterised as the presence of *external effects*. External effects play an important role in welfare analyses, because a perfectly competitive market can be shown to achieve an efficient allocation of goods only in the *absence* of external effects. In other words, from a theoretical perspective, any deviation from these conditions will result in either an under- or over-supply of the good under consideration, in this case land.

One of the consequences of the dominant position of the neoclassical framework in current microeconomic theory is that the presence of agglomerations is relatively difficult to account for. Many plausible explanations for the existence of cities are based on positive external effects, as an agglomerative force. In a somewhat simplified elaboration of this argument one could suggest that cities exist because people like to live close to each other. Although this may hardly count as a sound explanation, its relevance lies primarily in the implication that in neoclassical economics an agglomeration would never be considered optimal. And as far as the optimal solution is to be achieved, strict neoclassical economics ultimately can not explain the existence of agglomerations (Koopmans, 1957; Koopmans and Beckmann, 1957; Fujita and Thisse, 2002).

Similar arguments can be developed for land use in a broader context. Decisions on changing land use types, such as the conversion from agricultural to residential, do not only affect the level of well-being of the land owners or the people that will live in the new houses, but also other residents by altering the view of the landscape, traffic congestion or the introduction of a nearby shopping centre. Many of these effects are not allocated directly by markets and count therefore, likewise, as external effects. These types of external effects are usually accommodated in a neoclassical framework by pointing out that a government should intervene where markets fail. From the perspective of public sector economics, land use planning and monitoring the performance of land markets are therefore tasks that belong in the portfolio of a government, securing a fair distribution of welfare among its citizens. Whereas this second type of externalities can be assessed in terms of optimality using the neoclassical framework (Coase, 1960), the welfare effects of their interactions with agglomeration externalities are more difficult to interpret. More specific research questions regarding this issue are introduced and discussed in section 1.6.

1.2.3 Welfare in spatial equilibrium

From the perspective of public policy, land use planning would ideally be guided by a framework that would allow policy makers to compare different kinds of land use options using a single criterion. This criterion would incorporate economic, social and environmental effects of land use planning and allow policy makers to evaluate them using a single indicator. The search for a single indicator for policy evaluation is not uncommon in economic theory, as the decision between competing projects is often guided by a *cost-benefit analysis* (CBA). If the benefits from the non-market goods described in the previous sub-section are included in the CBA, the indicator should reflect the type of information required. Stated differently, this type of framework—as a foundation for a model or tool—should facilitate the user in conducting a *spatial social cost-benefit analysis*.

In principle, a combination of different sub-disciplines of economics might be a suitable starting point for developing such a framework. If economics is considered to study the allocation of goods in the first place and the institutions that perform the allocation in the second, the type of goods need not to be limited to those traded on a market. Various types of non-market goods might be included to cover environmental and social aspects of amenities and agglomeration externalities. Furthermore, if it is possible to derive an indicator that combines the various effects from land use as a spatial welfare measure, its optimal development over time might also serve as a proxy to the notion of sustainable land use. In resource economics and optimal growth theory, sustainable development is defined as the fair distribution of a societal welfare level over generations (Mäler, 1974; Dasgupta and Mäler, 2000). Following this tradition, a framework that employs a spatial societal welfare concept could in principle be extended to assess the dynamics of land use patterns. How the fairness of this distribution to successive generations can be assessed will need to be elaborated further, but the starting point would be a 'spatial social welfare function'.

In addition to the fairness of the distribution of welfare between generations, the *equity aspects* of a spatial welfare distribution can be assessed for a static solution of an allocation problem[1]. Especially because prices on a land market partly reflect the value of local characteristics, a tendency might occur in a region toward the exclusion of certain income groups from enjoying a socially—or politically—desirable level of environmental quality. If for example, from a theoretical point of view the negative side effects of air pollution are efficiently compensated by low housing prices, it can nevertheless be unacceptable for a society to confront only low income groups with this problem.

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[1] As will be argued in chapter 3, from a modelling perspective the word *stationary* is preferable over *static* in many situations. Here, 'static' refers to the solution in which time does not play a role.

1.3 Behavioural models

A cost-benefit analysis or a welfare analysis in general, relies on the comparison of the welfare levels in two or more different situations. If one is the current situation, the other situation or situations, will necessarily be counter-factual and a model is needed for exploring possible welfare levels and comparing them. Although this observation might seem rather obvious at first glance, it highlights the fact that even if prediction is not the main goal of a CBA, a formalisation of the future behaviour of relevant variables is still needed. The formalisation can range from an ad hoc extrapolation of the development of global market prices or economic growth, to a detailed model of individual consumer behaviour. Especially with models that contain assumptions about the behaviour of humans, as individuals or in aggregate, the results of a CBA require a careful interpretation.

The costs in a CBA can often be related to market prices. In many cases the behavioural model enters the CBA primarily in the estimation of the *benefits*. Benefits—or the improvement in welfare—can be expressed in a monetary value, either as the willingness to pay (WTP) or willingness to accept (WTA). The type of behavioural model applied in this context is essentially used for estimating the *demand* for a good—in most cases a public good (see 1.2.3)—in a counter-factual situation. A simple and practical example could concern the estimation of the benefits people experience from extending existing infrastructure with a new bridge. The bridge does not yet exist, hence the situation in which the bridge has been built is counter-factual. A behavioural model is needed to derive an estimation of the number of people that will use the bridge, once it is constructed. In an analysis similar to the way the demand for a normal market good depends on its price, the demand for a public work such as a bridge can be thought to depend on a (virtual) price, its value, using the same behavioural model. As long as a demand curve can be inverted, there exists a one-to-one correspondence between demand and price, or value, and vice versa.

In traditional approaches to CBA the evaluation of alternatives is usually based on an essentially neoclassical behavioural model for estimating benefits. This also applies to cases where non-market goods would be taken into account, as in environmental economics, following the tradition of Mäler (1974). Neoclassical here refers in a strict sense to the framework of Arrow and Debreu (1954) of which the conditions, in terms of the behaviour of economic agents, have been summarised as four axioms, or postulates, by Koopmans (1957, p. 53):

1. Non-increasing returns to scale for each producer,

2. A convex and representable preference ordering for each consumer,

3. Absence of interactions between any two production processes,

4. Independence of any man's preference structure from any production process and from any other man's choice.

If people accept prices as given under these conditions, it can be shown that the allocation of goods is *Pareto efficient,* meaning that 'nobody can be better off without making somebody else worse off' (Stiglitz, 2000, p. 57). This result is often restated as: Under the conditions stated above, perfectly competitive markets secure an optimal allocation of goods. As discussed in section 1.2, land policy is nearly always confronted with both positive and negative external effects, which implies the violation of one or more of the conditions above. From a public policy perspective, the framework needed for defining a spatial welfare level as the main indicator in spatially explicit social cost-benefit analysis cannot rely on the neoclassical framework for defining the counter-factual spatial equilibrium. The main reason is that a spatial welfare function will need to accommodate agglomeration externalities, even though the presence of externalities would suggest an inefficient allocation of land in the neoclassical framework.

1.3.1 Space in economic theory

Until recently, mainstream economics was essentially non-spatial. More precisely, in the neoclassical framework the locations where production and consumption take place are not specified. It can even be shown that in general it is not possible to extend the behavioural assumptions underlying the neoclassical framework without affecting the efficiency of allocation in competitive markets (Koopmans and Beckmann, 1957). The requirement that direct interdependencies between agents—other than those mediated by prices—are absent, reflected in postulate 3 and 4 cited above, is especially problematic in a spatial context. For example, most people tend to live in cities at least partly because of the opportunity to interact socially—and not only economically—with other people. Alternatively, if interaction is limited to market interactions *product differentiation* is required, which would violate postulate 1 (be discussed in chapter 4). The dominant position of the neoclassical framework within economic theory and its focus on perfect competition has let spatial economics develop in relative isolation from mainstream economics. Nearly the only connection between spatial and neoclassical economics was established by theories that can be traced back to the work of von Thünen (1826) at the beginning of the 19th century. These theories are generally limited to explaining land use patterns in a highly stylised way and presume the existence of a city centre instead of explaining its emergence. Other spatially explicit assessments of socio-economic topics between the 1950's and the 1990's were largely the domain of social geography, economic geography, and regional science.

Although it remains speculative, the isolation of spatial economics from mainstream economics, combined with the lack of spatially explicit assessments in neoclassical economics, might explain why some researchers have explored more or less radical—or *non-economic*[2] —alternatives to the neoclassical frameworks for explaining land use changes and other spatial phenomena such as the emergence of agglomerations (Haag, 1989; Allen, 1997; Nijkamp and Reggiani, 1998). The alternatives often include theories and methods that can be identified with so-called *complex systems* and range from neural networks to cellular automata and agent-based models. Although some of the research on complex systems already began in the 1950's and earlier, these concepts became very popular in the 1980's and 1990's. It is likely that with the exception of the economic interpretation of the concepts related to complexity in evolutionary and ecological economics, many of the original theories had no direct socio-economic theoretical content. Most concepts were applied in, or were at least inspired by, theoretical physics and mathematical biology. In this respect, they can offer a metaphorical explanation for the occurrence of spatial patterns but a comparison with behavioural and normative economic interpretations of the neoclassical framework is difficult. Applications to a public policy domain are often limited to straightforward prediction of for example, land use patterns, without a welfare—or comparable normative—interpretation.

The sharp distinction between mainstream economics and its radical counterparts seems to disappear in several developments that begin in the 1970s and crystallise in several publications that appeared mostly in the second half of the 1990s (see also Manski (2000) for a synthesis of several developments in economic theory). One development was related to the application of the Dixit-Stiglitz framework (Dixit and Stiglitz, 1977) of product differentiation to a two-region model by Krugman (1991) that lead to the establishment of the New Economic Geography (Fujita et al., 1999), for which—as some people argue—the name New Geographical Economics might be more appropriate. A development with a broader scope is sometimes labelled New Social Economics (Durlauf and Young, 2001), but is better known by the references to the specific topic of *social* or *non-market interactions*. In the context of spatial economics, these are often identified as neighbourhood effects (Durlauf, 2004). Both developments have at least part of their origin in game theory (von Neumann and Morgenstern, 1944) and its relation with dynamical systems in Evolutionary Game Theory (Weibull, 1995). In models where stochastic terms are added, the relation between discrete choice models (McFadden, 1973) and statistical physics can be explored (Durlauf, 1997; Brock, 1997). Evolutionary game theory provides a basis for the formalisation of interacting agents against the background of the mathematical analysis of complex systems that characterises many of

[2] See also Anas et al. (1998) for on overview of both economic and non-economic models concerning urban spatial structure.

the proposed alternatives to the neoclassical framework mentioned above. Statistical physics offers an analogy between individual agents and particles, while discrete choice models have a clear behavioural interpretation at the individual level. Finally, because a welfare analysis is possible for discrete choice models (Small and Rosen, 1981), this relation facilitates the interpretation of the physics-inspired alternatives in terms of traditional economics.

In accordance with the limitations discussed above, these models depart from nearly all the postulates of the Arrow-Debreu framework. This is perhaps most clear for postulates 3 and 4, in case non-market interactions are allowed for. Furthermore, the notion of increasing returns to scale is implied by product differentiation, because producers cannot afford to specialise while selling their products at marginal cost of production. It is unlikely that the clear separation of roles between market and state can be sustained if the neoclassical conditions are not met. Therefore a new welfare economic assessment is needed. Baldwin et al. (2003) present an example of investigating the welfare implications of the New Economic Geography frameworks, with a focus on regional economic development. This thesis focuses on the welfare assessment of land use patterns at the local level. With respect to the convergence of traditional and alternative frameworks, possible contributions of complex systems theories to this welfare analysis will also be explored.

Following a more general development in public policy, some national governments—including the Dutch—explore the possibilities of introducing more market forces and less regulation in the land market. This raises questions about the optimality of the resulting outcome. In the next subsection, this development is placed in a broader perspective of the reliance of market forces in the development of policy instruments.

1.3.2 Economic institutions

Most of the debates in the 20th century on economics and the division of tasks between market and state, concern the conditions for a price of a good to contain all information relevant for consumption and production decisions by all agents in an economy. Economic theory suggests that a market will secure the optimal allocation of a good if individuals make decisions only on the basis of these prices. In simplified terms, the role of the state here is twofold. It needs to make sure some goods that a market cannot supply, because no prices can be charged individually that would cover the costs, are supplied by the state itself; the so-called *public goods*. It also needs to prevent private parties from achieving monopolistic positions, charging prices much higher than the actual costs for producing the goods. An additional third task is related to the first two. If production or consumption affect the level of well-being by means other than price, the government has to design some kind of taxation system, making sure that the effects are translated into costs. An example of this type of regulation is the polluter pays-principle.

Questions were raised in the political arena beginning in the 1980s about the state performing tasks itself that might be dealt with better by a market. Following the conceptualisation above, the state would be a monopolist in this respect, overcharging and wasting tax money. For the allocation of some goods, such as telecommunication, energy supply, and public transport, quasi-markets were designed where private suppliers could operate under supervision of the state. In this way, the supplier would be able to charge prices that—due to competition—would be nearly equal to production costs as in a competitive market. The state would secure the public good quality of the supplied good, especially in cases where due to the lack of profit potentials the goods would otherwise not be supplied at all. An example of the latter is the subsidy for privatised public transport in rural areas.

More recently, the extent to which many countries in the 1990s adopted market-based approaches for the public sector is under debate. This issue primarily concerns the evaluation of the application of economic theory. At the beginning of the 21st century top American economists such as Joseph Stiglitz and Paul Krugman reviewed the 'roaring nineties' (Stiglitz, 2003). They concluded that the nearly unlimited belief in Adam Smiths Invisible Hand—or a certain interpretation of it—that guided the policies of privatisation in that decade, caused damage to many countries (Stiglitz, 2002, 2003; Krugman, 2003). The main argument is that conditions for markets to work were simply not met in reality (Stiglitz, 2003, p. 284):

> *'Underlying the views in favour of a minimalist government was a simplistic ideology, one I referred to earlier as "market fundamentalism", which said that by and large markets by themselves are both stable and efficient. I call it an ideology because it is a matter of faith: it rests on no acceptable economic theory, and is contradicted by a host of experiences (it would be true, for instance, if there were perfect information, perfect competition, complete markets, etc.—conditions that are simply not true in the most advanced of countries).'*

This observation holds in general, but is expected to be particularly relevant in a spatial context. The criticism of Stiglitz and other economists is therefore directed at the debate 'market vs. state' as a whole, instead of taking positions in it. As Stiglitz (2003, p. 305) states:

> *'The debate over the role of the government has, in recent decades, been broadened and enriched. There is clearly a need for collective action, but government is not the only way by which we act collectively.'*

Since the neoclassical optimal division of responsibilities between state and market depends on the validity of the postulates stated on page 16 section 1.3, the need for a different institutional arrangement arises in situations where one or more of the axioms cannot be applied. In a relatively abstract sense, this need also seems to be reflected in the redefinition of the role of the government in political science. This redefinition is often referred to as a transition from government to *governance*. As discussed above, often the axioms are inherently impossible to maintain in a spatially explicit economic setting. The reasons are primarily theoretical and only a few exceptions exist. In this sense, land use planning is a suitable topic for investigating a possibly more differentiated role for government.

1.4 Complexity

For several reasons, the difficulties of a positive interpretation of neoclassical economics are the subject of many debates within and outside the economics literature. Often perceived as problematic are for example, the assumed level of *rationality* and the lack of altruistic behaviour. However, if agglomerations are accounted for according to the recent literature in spatial economics, even market prices that would emerge in a system with self-interested, rational actors will not result in an efficient allocation of land due to the presence of external effects. If external effects are accepted as an existing direct influence of agents on other agents not only a departure from perfect competition can be marked clearly. With the external effects relabelled as *interactions*, this departure also marks the transition from a linear to a non-linear system that can be interpreted in a strictly formal way. As Koopmans (1957) showed, the conditions allow for a 'linear activity analysis' (Koopmans, 1957, p.67–68), or the use of *linear algebra* as the main tool for microeconomic analysis. This use is mainly characterised by the possibility to aggregate simply by means of *addition*. Mathematical tools for analysing interactions will have to rely on the ability of handling non-linear—or *complex*—systems. Often numerical computation—or *simulation*—facilitates a qualitative assessment of the overall behaviour of these systems. Nearly all the developments in traditional economics mentioned in section 1.3.1, resulting in the use of tools that were previously part of the radial alternatives, are related to the research on *complex dynamical systems* (Anderson and Arrow, 1988; Arthur et al., 1997; Blume and Durlauf, 2006).

While the word 'linear' in neoclassical microeconomics is essentially related to aggregation, the analysis is usually based on static equilibrium solutions. Using concepts from complex dynamical systems theories does not only require an interpretation of any non-linearities in the system, but also of its dynamics. As it appears, accounting for the dynamics is most effectively combined with addressing the topic of rationality.

1.4.1 Computer simulation

Perhaps one of the clearest manifestations of merging neoclassical and alternative approaches, in economics in general, is reflected in the sub-discipline of agent-based computational economics (ACE). Agent-based computational economics can be characterised as a ‘constructive’ (Tesfatsion, 2006) or ‘generative’ (Epstein, 2006) approach to economic theory. Instead of relying on mathematical proofs by means of explanation, ACE requires that observed regularities can be reproduced by means of simulation. Epstein and Axtell (1996, p. 20) suggest that if this requirement would be adopted in the social sciences, eventually the question ‘Can you explain it?’ might be replaced by the question ‘Can you grow it?’ Although methodological issues will be discussed in more detail in the next chapter, the concept of a ‘generative’ approach will serve throughout this thesis as a guideline for developing a framework in which the notion of spatial welfare can be defined. One interesting aspect of *generative social science* is the relation it establishes between *computational methods* (Judd, 1998) and what is sometimes called *social simulation* (Gilbert and Troitzsch, 2005).

Using a computer for solving equations is a well-established practise in many disciplines. Especially for models that cannot be solved analytically, computer simulations are often the only means for gaining insight into system behaviour and producing numerical results. ACE and the generative approach take the additional step of requiring that the numerical simulation corresponds to models of individual behaviour. In a neoclassical framework, due to the First Welfare Theorem, the outcome following the direct optimisation of social welfare by a benevolent social planner is theoretically equivalent to the result of all the decisions made by individuals in a competitive market. The key characteristic attributed to a market often identified as Adam Smith’s Invisible Hand, is that individuals only act in pursuit of maximising personal welfare, without the intention of maximising the welfare of the society as a whole. The assumption that an optimal allocation will emerge from a market has a weak justification. Most frequently, it is assumed that individuals deal fully rationally with complete information and therefore the optimisation of a welfare function is considered identical to the market outcome. This assumption is commonly applied in traditional applied economics approaches such as computable general equilibrium modelling.

Attempts to replicate market equilibrium by simulating individual decisions show conflicting results. Apart from the difficulties in the behavioural interpretation of the assumptions regarding the decisions made by individual actors in traditional models, the presence of interactions suggests that a market price in a land market does not necessarily reflect a socially optimal outcome. This situation is not uncommon in neoclassical economics, as long as market imperfections are concerned. However, since market imperfections are defined in the neoclassical framework on the basis of the same postulates of individual behaviour that cannot be sustained in spatial economics, in this thesis the behavioural context of the formation of apparently non-optimal prices in a spatial context will be emphasised. Computer simulations in general are expected to be a valuable tool in this respect.

1.4.2 Policy relevance

Land policy can be defined as the regulation of the land market (Ministerie van VROM, 2001). If, however, land use planning decisions cannot be guided by the traditional division of roles between government and market, policy instruments for achieving a socially optimal land use configuration are not clearly defined. In that case a *transition* is needed to an institutional framework that can accommodate a notion of spatial welfare, together with a theoretical framework in which policy instruments can be evaluated. The research questions will primarily be devoted to the theoretical framework. Nevertheless, based on the considerations of subsection 1.3, this theoretical framework is expected to offer suggestions for policy instruments and the accompanying institutional context. This institutional context might be identified with the concept of *governance*. In this thesis, however, its contrast with neoclassical economics is interpreted in stricter economics theoretic terms than is common in parts of the political science literature. Instead of confronting the theory on the efficiency of market allocation with the reality of policy implementation (see for example Bressers and Huitema, 1999), the economic-theoretical justification of adopting economic instruments in policy making will be examined. If a market allocation is not a priori efficient, because of the presence of external effects, the role of the state—or government—will consist of more tasks than the elimination of these effects. As far as ‘governance’ refers to more hybrid institutional arrangements than the dichotomous classification of market and state, taking into account interactions can be considered a starting point for a possible economic theoretical formalisation of stakeholder participation and collective action.

In a similar contrast with neoclassical microeconomics, the ‘complexity’ of the policy context can be identified with non-linearity in a strict correspondence with the mathematics of the framework, instead of a more metaphorical reference to a multi-actor environment. Allowing for interdependent individual preferences is likely to result in self-organising dynamics at the aggregate level of populations[3] possibly with multiple equilibria. These aspects of complex systems are especially relevant in regulatory policies for example, because of the possible discrepancies between the intended effect at the individual level and the final effect due to unintended response at the system level. Examining policy instruments in the dynamic context of a self-

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[3] Early examples of these phenomena can be found in Schelling (1978).

organising market economy offers the possibility of exploring how a government can deal with complexity.

1.5 Aim

This thesis studies the effect of land policy on social welfare, taking into account the presence of market and non-market interactions that introduces various degrees of complexity in the land use system. For this purpose a model will be developed based on the Alonso model of urban economics (Alonso, 1964). The original model, however, will be adapted to allow for an evolutionary approach to the formation of land use patterns. Furthermore, the evolutionary variant is more general than the original Alonso model, as it can accommodate several types of external effects.

1.6 Research design and research questions

Traditional economics of the public sector in a non-spatial context applies a normative framework on which instruments are based to achieve an optimal level of social welfare. A projection of a similar concept regarding spatial planning would naturally employ a notion of socially optimal land use. The policy task for a government could be defined as finding a balance between various functions or land use types, taking into account use and non-use of land. The first concerns urbanisation and infrastructure, the latter can be intended to preserve future use or biodiversity. Intervention by a government would in traditional public sector economics depend on the public good nature of for example, open space. However, a public good in the neoclassical framework is only defined against the background of a market with perfect competition. If the neoclassical postulates for individual behaviour cannot be applied in a spatial context, socially optimal land use should probably be defined beyond the traditional distinction between public and private goods. In that case, a new reference needs to be designed to evaluate the effects of policy instruments.

1.6.1 Research questions

The First Theorem of Welfare Economics states that, if neoclassical behaviour applies, a decentralised allocation is Pareto-efficient. This theorem is often projected on a competitive market. It suggests that if goods are allocated by means of prices and supply equals demand, ‘nobody can be better off, without making somebody else worse off’ (Stiglitz, 2000, p. 57). When translating this theorem to a land market, two types of considerations must be taken. The first type is related to the observation, referred to in section 1.2, that in many cases the neoclassical framework can not be applied in a spatially explicit context, because of economic theoretical reasons. The second type of considerations is more traditional. From a public sector economic perspective, a welfare economic assessment of a land market deals not only with the efficient allocation of land, but also with the allocation of its characteristics. In a first approximation, these characteristics could be interpreted as the local (environmental) quality and would therefore comply to the definition of a local public good, because they count as exogenously given for market parties, but do affect their level of well-being. This aspect is most apparent in public amenities, such as city lights and trees in streets. If, in a first assessment, the quality of a location would indeed be considered a local public good, two goods would be traded simultaneously on a land market:

1. land, as a market good and,
2. quality, as a local public good.

From a public policy point of view, an ideal welfare measure would address both aspects simultaneously. In this stylised case, public policy is confronted with two aspects of a socially optimal allocation of land:

1. securing optimal allocation of land by markets and,
2. securing an optimal distribution of local public goods, or amenities.

The presence of amenities, however, has an impact on the price of land. Their value, or at least part of it, is said to be *capitalised* in the price for land and the effect of capitalisation on the efficiency of the allocation of land needs to be taken into account.

In addition, it might be argued that the role of the government goes further than securing an efficient allocation of land with its amenities. Open space can also be considered an amenity. As a public good, it would typically be under-supplied by markets and the state would be called to intervene. The protection of open space could in this perspective be interpreted as the direct supply by the government of a good that contributes to the well-being of all consumers, but which cannot be allocated efficiently by markets. However, in terms of land use open space is an alternative use to other land use types, such as residential space. Even if open space is interpreted analogously to a natural capital stock, this ‘non-use’ type competes with other land use types. Therefore, a third aspect of a socially optimal allocation of land will be added:

3. securing an optimal allocation of land without immediate social-economic use, designated for habitat protection or open space as a public good.

These theoretical considerations lead to the first research question:

> *Question I*
>
> *How can traditional public sector economic concepts concerning a socially optimal outcome such as efficiency, optimality, and equity be translated to land markets?*

In section 1.4, also the observation was made that in recent developments in spatial economics and in the development of interaction-based approaches in economics in general, aspects from *complex systems theory* seem to be combined with more traditional normative economic—nearly neoclassical—interpretations. Given the special role these methods have in spatial economics, by representing externalities, this role will be articulated for the land market in the second research question:

> *Question II*
>
> *What is the role of complex systems theories in the formation of land prices and how does it correspond to, or differ from, price formation in neoclassical markets for consumer goods?*

As noted in section 1.4.1 especially the *generative* approach of agent-based computational economics is expected to be beneficial in answering this question. The identification of a market outcome with a generative approach facilitates defining the role of governments and designing policy instruments. While traditional optimisation methods can be used to calculate the optimal welfare level that can theoretically be achieved, the impact of policy instruments at the level of individuals can be explored and verified by using agent-based simulation models.

The third and final research question aims at the translation of the answer to the previous question to practical policy recommendations. In public sector economics, the normative quality of the neoclassical framework serves as an ideal benchmark. This commonly introduces concepts such as market failure and government failure. However, if a socially optimal outcome and market dynamics need to be redefined for land markets, in principle, actual policy making should be empirically validated against this new benchmark. Besides, possible policy instruments for improving 'spatial welfare' will need to be defined relative to it. This is a question regarding the possibilities for operationalisation and is therefore—although considered as a precondition for a transition toward sustainable land use—relatively modest in its ambition:

> *Question III*
>
> *To what extent can the spatial equivalents of welfare economic concepts be applied in policymaking, concerning land use planning?*

This question focuses in particular on the expected departure from the traditional dichotomy market-state and the possibilities for governance structures that might contribute to a future transition to sustainable land use planning. Suggestions and directions for the development of new policy instruments are expected.

1.7 Structure

The structure of this thesis is as follows. Chapter 2 is devoted to a methodological overview of two ways to model the behaviour of humans. The first is methodological individualism, the second concerns systems approaches. Moreover, it is shown how mathematical biology can offer a conceptual basis for integrating the two approaches. In chapter 3, two types of complexity are selected that are relevant for this thesis. The first is derived from non-linear dynamical systems. The second type concerns *computational* complexity. Both types of complexity will be related to the issue of interactions and aggregation against the background of the linear activity model in neoclassical economics. In chapter 4, the interpretation of both types of complexity and their relation with neoclassical economics from chapter 3 is clarified with the help of a simple two-agent, two-goods exchange economy model.

The literature on spatial economics is reviewed in chapter 5, with special attention for land use, welfare and complexity. These issues return in chapter 6, where the two-agent model of chapter 4 is used to generalise the Alonso model of urban economics as a population game. The model from chapter 6 is applied in chapter 7 for welfare analyses concerning land use, with a focus on the welfare effects of open space. In chapter 8, the generalised Alonso model is converted to a full-fledged multi-agent system (MAS), especially designed to incorporated models of discrete choice with interactions. With this type of MAS, labelled Multi-Agent Discrete Choice Model (MADCM), more specific simulation runs are conducted, that explore the effects of a wide range of external effects, path dependency and imperfect information. Chapter 9 presents a summary and conclusions.

Chapter 2

Methodology

2.1 Introduction

In section 1.3 the observation was made that any framework for conducting a welfare analysis is likely to contain assumptions about the behaviour of humans. This chapter first explores the methodological issues of modelling human behaviour at the relatively high abstraction level of the differences between individual-based versus systems perspectives. Following the ideas developed earlier in essays by Koopmans (1957) and Epstein (2006) it will be argued that formal frameworks in the social sciences—with the exception of statistics—are essentially deductive, as they can be read as the implications derived from a set of axioms, or *postulates*. Furthermore, using a deductive framework in the social sciences is usually constrained by the fact that, unlike in physics, implications derived from postulates do not necessarily have a one-to-one correspondence with empirical observations. This marks an important difference with the natural sciences, highlighted by the possibility in the social sciences of choosing one of several formal frameworks. Therefore, a criterion is needed for judging the explanatory value of a model.

An attractive criterion based on computer simulations was introduced in *agent-based modelling* and more specifically in *agent-based computational economics* by Epstein and Axtell (1996) (see also Epstein, 2006). Assumptions concerning the behaviour of individuals are specified in several types of formal models. If these assumptions are translated to *behavioural rules* in a computer simulation model, the *possibility of replicating* a stylised version of an observed phenomenon with that model might be considered as a way of understanding it. This approach is contrasted with a mathematical proof of the existence of a solution to a given problem. An existence proof requires additional assumptions about how human beings would be able to find the solution. Furthermore, if a simulation model is able to replicate the occurrence of an observed non-optimal situation, the behavioural rules in the model might offer a starting point for improvement, for example by means of public policy. The role of computer simulation methods in both individual-based and system approaches are discussed in section 2.6.

A third methodological topic will be discussed in a separate chapter, chapter 3. It concerns the role of *complexity* in formal models. Especially the issue of *non-linearity* in relation to neoclassical economics raises issues that directly affect the research questions in section 1.6 centred on welfare economics. It was noted that the neoclassical framework relies on very stringent behavioural assumptions, while in relatively recent developments in economic theory models of imperfect competition are employed, adopting elements similar to those from frameworks that previously had been introduced as more or less radical alternatives to neoclassical economics. A more detailed discussion on the application of these models in spatial economics will be postponed until chapter 6.

2.2 Behavioural models in the social sciences

The use of models in the social sciences naturally focuses on modelling human behaviour. There appears to exist a variety of frameworks for this task and the researcher needs to make a choice. Up to the extent that in the social sciences any modelling framework represents a certain theory of human behaviour, different theories can apparently be applied for modelling the same phenomenon. This situation characterises the social sciences in general and marks a major difference between the social and the natural sciences in connecting models and theories. In the latter, there exists a close correspondence to an accepted theory and a formalisation that can be used for modelling specific cases.

An often used heuristic definition of a model expresses its interpretation in terms of a simplified representation of the real world, or at least of elements from it. Because the selection of the elements might be considered an act of simplification, the definition ‘simplified representation of reality’ is sufficient for the discussion in this section. A model describes the relation between the selected elements. In the natural sciences, this description is nearly always expressed in mathematics. Mathematics as the main tool for modelling also distinguishes the natural sciences; the social sciences allow for less rigid descriptions in a natural language or loosely sketched schemes to be called models as well. Nevertheless, in some disciplines in the social sciences, mathematical models have been developed that adopt a similar approach as the models in the natural sciences. Occasionally, in some parts of the social sciences the claim is stated that *computer models* represent an alternative to mathematical models. This claim will be discussed in section 2.6.

If the set of models is restricted to mathematical models, the question arises of how a collection of mathematical equations might represent reality. This is easier to answer for the natural than for the social sciences. Especially in the field of applied physics and engineering, the laws that govern the conservation of momentum—Newton’s *Laws of Motion*—, the conservation of mass and the conservation of energy are the basic ingredients for any model. For example the basic set of equations in fluid dynamics, the Navier-Stokes equations, from which many models are derived—ranging from engineering applications to fundamental research—, can be considered a reformulation of basic conservation laws. These conservation laws can be thought of as regularities that can be confirmed empirically. They can be expressed efficiently in mathematical equations that contain variables that correspond to measurable quantities, such as velocity, pressure and mass. In this sense, most engineering models could essentially be considered a restatement of conservation laws. This observation has severe implications for the heuristic definition referred to above. If the conservation laws themselves need no empirical testing, any mathematical model based on these laws essentially represents elements of the real world—at least in a way that is sufficient for practical purposes, such as building a bridge or plane.

Simplification in the natural and engineering sciences often means a reduction of variables, laws, or dimensions, if convenient. In many applications a model of a fluid is still applicable if it is assumed that the fluid is incompressible and two-dimensional. A model of an incompressible flow can largely do without considerations about the conservation of energy, which then simplifies the model. A two-dimensional flow might be considered as a cross-section, where variation in the direction perpendicular to the cross-section can be neglected in a first approximation. Forces on the wing of a plane can in this way be calculated at first in only two dimensions, which would correspond to a model of a wing with an infinite length. Although the simplification in an engineering model above all concerns leaving out some variables, laws or dimensions, the laws themselves are not considered simplifications of reality in the first place. Of course, the conservation of momentum—as a restatement of Newtonian physics—itself is a simplification, as relativity theory and quantum physics show. However, if the model is used for a description in an environment where Newton's laws can be applied, it is likely to contain nothing more than a restatement of these laws. For a model of a brick that is thrown from a tower, Newtonian physics is still accurate enough.

There exists no equivalent to the conservation laws in the social sciences; there are no 'Fundamental Laws of Human Behaviour'. Perhaps the closest social science ever approached the natural sciences from a methodological perspective is contained in the formalisation of the homo economicus, or *rational actor*. Unfortunately, the success of the rational actor model in representing reality is rather limited. Empirical results from experimental economics strongly suggests that people often behave differently—especially in situations derived from game theory—than what the theory would predict. A popular example in this context is the Prisoner's Dilemma and related games. In that respect, the heuristic definition of a model above seems more accurate in the social sciences, because the rational actor can be considered a simplified representation of a human being. Although the validity of the model is often empirically refuted, there does not exist an equivalently general model that yields better results, perhaps only for special cases. This observation, however, is consistent again with the absence of laws in social science, though there is some regularity in human behaviour and models might help in understanding them. Regularities can be observed and measured by using statistical methods. Quantification follows in terms of probabilities and frequencies, but statistics does not specify any behavioural rules. If, as in econometrics, probabilities are used for estimating the parameters of a behavioural model, the methodology of the natural sciences seems to be approximated as close as possible. However, a quantification of a 'goodness of fit' is usually needed to verify the validity of the assumptions of the behavioural model. The need for this type of test shows that a social scientist is still confronted with a multitude of models. As a consequence, unlike many natural scientists, the social scientist needs criteria for deciding which model provides the ‘best’ explanation for an observed phenomenon.

The procedure of working backward to theory from observed regularities is called *induction* in the philosophy of science and in methodology. It applies mostly to statistical and econometric methods. The methodological assessment of estimating parameters in a specified model, as in econometrics, is primarily concerned with the issue of identification (Koopmans, 1949; Manski, 1995; Hoover, 2006). Although a theoretical model might distinguish between two or more effects, it is not always possible to assign parameters to these effects in an unambiguous way using data from observations. Two people might, for example, live in the same neighbourhood because they both like the area characteristics independently, or because both happen to belong to the only income group that can afford buying a house there, or because they prefer to live close to each other. If the only observations consist of personal characteristics, location characteristics, and the location choice, it depends on the specification of the location choice decision whether the model can distinguish between the three possible explanations mentioned above. The need for criteria is also present when the researcher tries to explain a phenomenon for which he does not use numerical data, but observed stylised facts. Examples of this type of modelling approach can be found in urban economics, where the existence of cities might be interpreted as a stylised fact and an explanation is sought in the economic behaviour of individual agents.

Although an explanation or a theory constructed in this way can resemble a law of natural science, its validity is nearly always less universal in the social sciences. The explanatory value lies foremost in *deduction* and depends entirely on the assumptions from which the conclusions can be derived. Whereas with induction theory is derived from observation, with deduction theory is derived only from reasoning and logic. The assumptions therefore have the status of axioms or postulates for which no explanations are given in the theory. As a consequence, criteria other than strict empirical verification apply for evaluating a deductive theory. In general, a theory is considered to give a good explanation if a minimal number of more or less simple postulates are sufficient to explain relatively complex phenomena. The three laws of Newtonian physics have a similar status, but since observations always follow the implications that can be derived from these laws by deduction, criteria for an evaluation of these laws as postulates are obsolete. A theoretical model in the social sciences will need different criteria with regards to the plausibility of a suggested explanation for an observed phenomenon.

Deductive systems can be formalised by means of mathematical logic. In this sense, mathematical economics—and the use of mathematical models in economic theory in general—can be regarded as the formalisation of deductive reasoning. A prominent example is the Arrow-Debreu framework of general equilibrium (Arrow and Debreu, 1954; Debreu, 1959). It is often criticised by both economists and non-economists for its unrealistic assumptions concerning human behaviour. It serves nevertheless as an important reference and benchmark for other frameworks. This applies to economic theory, but also to other social sciences, for example sociology (Coleman, 1990). The common characteristics of theories related to the neoclassical framework can be identified primarily with the Arrow-Debreu framework of general equilibrium:

- the rational actor and,
- methodological individualism.

To organise the discussion of neoclassical economics and its alternatives, following Gintis (2000, p. 43), the label 'neoclassical' will be reserved exclusively for the Walrasian general equilibrium model. In many proposed alternatives, specific elements deviate from the neoclassical assumptions in order to propose a contrasting framework that is intended to yield more realistic results. Frameworks that change elements concerning rationality, while keeping an *individual-based* perspective, in general present a less radical alternative than frameworks that start from a *system* perspective. Both will be discussed in the following sections.

2.3 Individual-based methods

In the neoclassical general equilibrium model, agents are assumed to make decisions concerning their consumption only on the basis of market prices. This decision is rational in the sense that the agent chooses the consumption bundle he or she prefers over all other bundles. Although this preference structure is usually formalised by a utility function, the decision might alternatively be stated as a set of decision rules. Rather than the decision rules themselves, information about the various options the agent is supposed to have and her ability to process information usually pose the actual problem for the modeller. Concerns about the assumed level of rationality often apply to problems about information, instead of the ability to make decisions. In the following chapters the convention will be adopted that agents are rational in the sense that in principle they can distinguish the best option out of a limited set, based on either subjective beliefs or public information. Because the decision rules express subjective preferences, this notion of rationality is nearly meaningless. The only assumption necessary is that an agent is able to rank the options herself according to her own preferences. If she happens to prefer classical music over rock music, she will choose classical music. The issue of information availability and the ability of process information are more complicated. For the moment, only the minimal assumption will be made that the agent has a system of beliefs sufficient for making decisions. A stronger assumption might for example require that the agent's belief system results in a consistent ranking of options. Apart from the behavioural assumptions, the decision problem of the consumer in neoclassical economics is limited to that of consumption, with information flows reduced to market prices.

A much broader set of decision problems is addressed in *game theory*. Again, regardless of the behavioural assumptions made in some branches of game theory, the decisions agents make in game theoretical models are related to strategic interaction. Many types of interaction between human beings—and also between animals—can be thought of as 'strategic' at some relatively general level. As long as at least two agents need to make a decision and their preference structures are interdependent—meaning that the individual ranking of options by one agent depends on the choice the other agent is expected to make—, there exists a case of strategic interaction. This loose definition of strategic interaction therefore includes nearly every type of human interaction, as long as choices are characterised by dependency and coordination. It allows for cooperation, but does not assume it beforehand. In *non-cooperative game theory* self-interested behaviour is assumed. Picking a date for an evening among friends could be considered as problem that might be cast in the game theoretic setting. The preference of one agent for a specific evening clearly depends on the evening they are expected to choose.

Game theory is studied and applied in sociology, political science, mathematics, biology, and economics. Within economics it is studied in more general settings than just markets. Because game theory itself covers such a broad set of interaction types, the subset of interactions that involve markets can therefore be thought of as 'economic' in a stricter sense (Vega-Redondo, 2003). Usually, markets where agents can strategically exercise power are contrasted with competitive markets. Markets are considered competitive if the agents consider prices as given. In this sense, the framework of neoclassical economics primarily addresses non-strategic behaviour. Markets in which a certain number of, or all, agents can strategically influence the market prices are either not covered in neoclassical economics or are treated as an exception. This has two implications for the classification of theories within social science in general. First, markets that allow for strategic interaction by means of direct influence of market price formation—such as oligopolies—can be addressed in a different framework, usually that of game theory. The second implication is that any other form of interdependency complementing a competitive market can be labelled non-market interactions, since by definition in a competitive market agents base their decisions on the market prices as the only source of information. The contribution of non-market, or social interactions, can also be studied in a game theoretical framework.

The distinction between market interactions and strategic or non-market interactions is important with respect to the beliefs agents are required to have regarding their environment. Prices can be observed and interpreted by agents with relatively simple cognitive capabilities. In case of strategic interaction, agents will need to have a belief regarding the belief of their opponent. Information on the belief of an opponent is far more difficult to obtain than information on prices. Furthermore, for two agents to reach an agreement, their beliefs regarding the belief of the other agents will need to be consistent. As a consequence, one agent will need to assume that the other agent knows that the first agent knows that she knows, etc. This consistency aspect of rationality in game theory may be considered as most problematic. As far as rationality would only refer to the ability of an agent to decide what is best according to his own beliefs, it can be interpreted as a minimal requirement for the behaviour of any agent. For purely pragmatic reasons, it seems problematic to assume that agents choose options they do not consider as the best. This is not to argue that in reality people never act irrationally, even if the definition of rational behaviour would be restricted to making the subjectively best choice. Instead, a model of consistently irrational behaviour would lead to a paradox, unless the decisions are made purely randomly. And a model of purely random behaviour would have little relevance for research and policy applications.

At first, the restricted notion of rationality in the above sense seems more compatible with market interactions than with strategic interactions. Market prices can be assumed to be publicly available, which leads to a simpler belief representation than if beliefs had to represent the beliefs of others. Unfortunately, this conclusion cannot be drawn so easily because the process of price formation does in principle still suggest some kind of strategic interaction, such as bargaining. In neoclassical economics, it is often implicitly assumed that an external metaphorical institution—called the Walrasian Auctioneer—will determine market prices. This assumption is problematic in several respects. It transfers the information about all relevant beliefs from the agents to the Auctioneer. Additionally, assuming an external institution seems at odds with the suggestion that a neoclassical market is a representation of Adam Smith's 'Invisible Hand'. Smith wrote in his 'Wealth of Nations' (Smith, 1999, p. 32):

> *'He generally, indeed, neither intends to promote the public interest, nor knows how he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote and end which was no part of his intention.'*

Besides debates on the positive interpretation of the assumptions of the neoclassical behavioural model, its normative use is also a subject of criticism. The normative use of neoclassical economics can largely be identified with the political, ideological, adagio of ‘more market and less state’. However, this discourse appears to take place mainly within the economic discipline itself. The reason is most likely the rather technical nature of the discussion. Nevertheless, its policy implications can be far-reaching. The First Theorem of Welfare Economics states that if the conditions of neoclassical behaviour apply, a competitive market will yield a Pareto efficient allocation of goods. It suggests that if goods are allocated by means of prices and supply equals demand, ‘nobody can be better off, without making somebody else worse off’ (Stiglitz, 2000, p. 57). At first, this result may be considered a mathematical proof of the existence of Adam Smith’s Invisible Hand. However, since the neoclassical framework does not define the process that will lead to the optimal allocation, its normative claim cannot be entirely separated from its positive interpretation.

The lack of a process description in the neoclassical framework highlights the problematic position of a third characteristic of the neoclassical and related frameworks frequently discussed in scientific debates, that is,

- the focus on equilibrium solutions.

If the set of prices that correspond to an optimal allocation is interpreted as a *stationary* equilibrium, the Invisible Hand seems to refer to notions of *adaptive systems* and *self-organisation* rather than a *static* equilibrium. These concepts are usually identified with *system theories* and are the subject of the next section.

2.4 System theories

Taking the individual as the main element or building block of a modelling framework is not the only way of constructing a model in the social sciences. The element of choice referred to in the introduction of this chapter, allows the researcher to opt for methodological holism, or a *systems approach*, instead of methodological individualism, or the *individual-based approach* used in neoclassical microeconomics. In a more general sense, this issue concerns the question of whether social phenomena are modelled at the aggregate or the individual level. Macroeconomics can often be considered a systems approach, as far as micro-foundations are not accounted for. However, in this chapter the term ‘systems approaches’ is reserved for a specific type of models.

When looking at the mathematical modelling practise, it appears that the word ‘systems approach’ is usually reserved for systems of differential (and sometimes difference) equations. Especially in some fields of social science, the concept of ‘System Dynamics’ (Forrester, 1961) seems dominant as the systems approach. System Dynamics can be considered a subset of the all the elements in the more general mathematical theory on dynamical systems. As such, the concept of a system seems void of any meaning, because mathematics only deals with numbers and logic. The subset used in System Dynamics, however, can approximately be specified as the dynamical systems found in *control theory*. Control theory mainly has its applications in electrical and mechanical engineering. A very important concept in control theory is *information feedback*. For a device to keep temperature a prescribed level, heating or cooling can be done more accurately if it depends explicitly on the current temperature. Although this example might seem rather obvious, the mathematical formalisation of the accuracy is relatively new. Most of its work was done during World War II, applied to radar systems and ground to air missile systems. A lack of accuracy is often described in terms of overshoot and oscillating behaviour. These terms became rather popular is some social scientific fields after the publication of the work by the Club of Rome (‘Limits to Growth’, Meadows et al., 1972) based on the Systems Dynamics approach developed by Forrester (1961). More elaborate translations from control theory to social science can be found in Cybernetics, starting with the work by Wiener (1965). Both Forrester and Wiener themselves were involved as engineers in the research and development for missile guidance systems during World War II. A systems approach is also influential in some fields of sociology, following a more qualitative interpretation by Luhmann (1984). Finally, a systems approach as an interdisciplinary science was founded by Von Bertalanffy in 1951 as the General Systems Theory (von Bertalanffy, 1973).

The characterisation and interpretation of a systems approach remains difficult, even if the systems of differential equations are restricted to systems that bare some resemblance to the description based in control theory. The set of systems of differential equations that contain feedback loops can be made arbitrary large, but any notion of interdependence essentially reflects a feedback loop. The main difficulty with the interpretation of a systems approach in the social sciences is therefore neither its descriptive accuracy nor its lack of it; rather, it still does not supply the researcher with a theory of the behaviour—analogous to ‘motion’—of humans. It does not even provide a theory on the behaviour of systems as other entities or aggregates of human beings. Systems theory is not a theory in the usual sense. That is not really a surprise; it was already mentioned that the theory of dynamical systems is a mathematical theory; it only deals with reasoning not with behaviour. The mere formalisation in terms of differential equations does not address the issue of aggregation.

A second difficulty is philosophical rather than methodological. Any description of the behaviour of a system without explicit reference to the elements out of which it is constructed raises questions about the intentions of the system as an entity. Even a relatively mild assumption concerning the presence of a negative feedback can be interpreted as a teleological necessity of attaining an equilibrium state. This is probably best illustrated by the Gaia metaphor, introduced by Lovelock (1979). The suggestion that Earth itself is an organism is very vulnerable to the attack that it is unscientific to suggest that the Earth acts purposefully.

Notwithstanding the difficult interpretation of a systems approach as a general theory, the results from research on *complex systems*—as far as it is restricted to the mathematical theory of connected dynamical systems—have had implications for other theories as well. The combination of feedback mechanisms and non-linear differential equations gives rise to several complex phenomena. Especially the notion of *self-organisation* has attracted the attention of disciplines that try to explain regularities difficult or impossible to account for with linear models (Strogatz, 1996). A system perspective is able to accommodate many features of the so-called *complex systems theory*. On the other hand, complexity is occasionally identified with *interacting individuals*, which does not exclude an individual-based approach, or methodological individualism. While some success has been achieved in the operationalisation of non-linear systems in explaining regularities in physics, the most successful discipline in this respect is arguably biology. Perhaps because biology also deals with behaviour, some economists seem to follow ideas first developed in *mathematical biology* for introducing elements from complex dynamical systems in economic theory.

2.5 Mathematical biology and game theory

The previous subsections showed that system approaches can in principle address the complexity of the behaviour at the system level as a result of the feedback mechanisms between its subsystems. However, the complexity that results reflects only the properties of the mathematical description, while assigning behaviour at a system—or aggregate—level is difficult to separate from a teleological interpretation. Individual-based approaches on the other hand generally suffer from the problematic interpretation of mutually consistent beliefs of interacting agents.

A closer inspection of the fields where applications of a systems approach have been very successful, mathematical biology (Murray, 1990) and mathematical ecology (May, 1974), might resolve this issue at least partly. While a mathematical model of an ecosystem has some properties at an aggregate level, such as stability, these are usually the result of the well-specified interaction between several species. A famous example is the predator-prey cycle that can be characterised as a special type of oscillator, while the interpretation of the birth and death rates of the predator and the prey population can—in principle—be translated to individual members of both populations. The cyclic behaviour was independently formalised by Lotka and Volterra (May, 1974).

Lewontin (1961) was one of the biologists that introduced game theory for analysing population genetic theory, extending the seminal work on the statistical approach to population genetics by Fisher (1930). Maynard Smith (1974) and Taylor and Jonker (1978) applied equilibrium concepts from game theory to the analysis of conflict between behavioural types within a population. Instead of the direct interaction that consists of eating and being eaten in the Lotka-Volterra system, the models by Maynard Smith (1974) and Taylor and Jonker (1978) were based on the evolutionary advantage of adopting a certain behavioural strategy. A popular example contains the behavioural types ‘hawk’ and ‘dove’, not the species, but aggressive and peaceful behaviour. Behavioural types can be mutually exclusive or can coexists, depending on the distribution of the evolutionary advantages. Although some games of this type can show oscillating behaviour, the majority converge to a stationary equilibrium that can be interpreted as equilibrium in *fractions* of the population, similar to equilibrium in (mixed) strategies between two interacting agents in economics. Within both biology and economics, the analysis of these types of models is referred to as evolutionary game theory (EGT) (Hofbauer and Sigmund, 1988; Weibull, 1995; Hofbauer and Sigmund, 1998).

The formal interpretation of the equilibrium concepts in evolutionary game theory will be addressed in the next chapter. First, the attractiveness of population games at a more conceptual level is stressed, as it seems to suggest a solution for the both the system interpretation of complex dynamical systems and the interpretation of individual beliefs in decision making. On one hand, dynamical systems in evolutionary game theory have a clear reference to individuals in a population. The interpretation of a fraction of a population only has a strict mathematical justification in the assumption that the population is very large, but as an approximation this interpretation is often valid more generally. The notion of evolution, on the other hand, offers an interesting alternative to the consistent belief systems of interacting agents. Admittedly, this notion is relatively abstract, but for example the evolution of institutions and conventions might be interpreted as the result of a repeated game (Young, 1998), where people have adjusted their strategies over time. Whether people in a country drive on the right or left-hand side of the road is not important; most important is the agreement on one of the two sides. Although, the agreement can be interpreted in terms of strategic interactions, the problem of consistency in the beliefs of all agents on the road is delegated to legislation that has evolved over time. It is therefore only rational for the individual agent to obey the current convention and no extreme cognitive capabilities will need to be assumed.

2.6 Simulation methods

The word ‘simulation’ carries several meanings in relation to modelling. A common element is the reliance on a computer for model implementation. The more conservative meaning follows physics and engineering where simulation is often identical to numerical computation. Given the extensive use of differential equations in the natural and engineering sciences, often both simulation and numerical computation can read as synonyms for numerical integration of mathematical models of dynamical systems. The use of a computer becomes necessary if the system of equation cannot be solved analytically, deriving a closed form solution with pencil and paper. In the natural sciences analytical solutions are often preferred, as they suggest a more precise solution. Analytical solutions are also usually preferred in economics, though for a slightly different reason. In economic theory an analytical solution conforms to a deductive proof, whereas a numerical solution can only represent a special case.

A different approach to numerical integration is presented by *Monte Carlo* simulation or integration methods (Judd, 1998), often applied in econometrics. Simulation in econometrics has a slightly different meaning. There, it mainly concerns the estimation of the mean of a variable, relying on integration of this variable times a probability density, by calculating the average value on the basis of a very large number of draws from the probability density distribution. Simulation—by means of direct integration—in case of a dynamical system, usually concerns the reconstruction of a position or volume of an entity, given a description of its velocity or growth rate, using small but finite approximations for infinitesimal small quantities in the analytical equivalents.

Since systems of non-linear differential equations can not be solved analytically in general, with only a few exceptions, numerical integration of these systems is necessary for understanding their behaviour. A qualitative first impression of the overall behaviour of non-linear systems can often be supported by analytical solutions for (local) linearisations, especially concerning stability. However, a full account of the behaviour can only be given by solving the equations numerically. The integration of 2D and 3D visualisation in the graphical user interface (GUI) further facilitates the interpretation of the results. If a dynamical system is integrated over time, while the results are presented graphically and instantaneously at every calculation step, the computer approaches a virtual laboratory where experiments can be conducted.

Models based on dynamical systems are less common in economics than in physics. Instead, many economic models are based on a maximisation problem. Numerical maximisation can hardly be called simulation, since the algorithm is often designed to solve a problem without making any reference to a behavioural model. An exception can be found in the relatively recent development of genetic algorithms. Although the behavioural model is rather an abstract implementation of mutation and selection, it presents an approach closer to simulation, at least at the level of genes. Similar to evolutionary algorithms and even more strongly related to evolutionary game theory is a specific modelling approach in which models are formalised only in computer code. Once the decisions is made to model system phenomena only through a representation of the behaviour at the individual level, a next step might consists of taking advantage of an object-oriented computer language. This approach adopts concepts also applied in computer science (Weiss, 1999; Wooldridge, 2002). Although these so-called *agent-based models* can in principle be programmed in a procedural language as well—executing one command or function after another—the main benefit of an object-oriented (OO) language comes especially with the concept of *encapsulation*.

In any OO language the focus is on objects in which data are stored. These data are often called attributes. If a data-object represents an employee, one of his attributes could be his wage. The concept of encapsulation only refers to the possibility for the programmer to store the attribute in the object so that it is 'hidden', so that it forces the programmer to adopt the discipline of defining functions—or 'methods'—for accessing the attributes. The methods are interesting for modelling human behaviour, because they relate information to communication. In this way, agents will only reveal their information when asked to do so explicitly. If two agents meet, they might be able to 'ask' for information to each other. The information received can be stored in an attribute, which might be interpreted as 'belief'. Through a rather elaborate procedure, a dialogue between two agents, discussing their wages might be formalised as follows:

1. Agent A asks agent B to reveal her wage,
2. Agent B retrieves her wage level from the information only she has access to,
3. Agent B reveals her wage to agent A,
4. Agent A stores the information he receives from agent B in his 'belief system'.

Presented in this way, the benefits from this modelling approach are not immediately apparent. Instead of offering a more efficient way to modelling, it even seems to pose additional restrictions. However, in terms of a behavioural interpretation, these restrictions offer a transparent way of dealing with the problem of consistent beliefs in game theory. Beliefs can only become consistent in a more convincing way if they can be reconstructed with an elementary communication protocol. Agents can only become aware of the information other agents posses if they are able to 'ask in person'.

The idea of convincingly reconstructing a certain result lies at the heart of Agent-based Computational Economics (ACE). Epstein (1999, 2006) even explores the implication of posing it as a requirement:

> *'If you didn't grow it, you didn't explain it.'*

Interestingly, similar considerations have been discussed within the context of using computational methods in economic in general, not only restricted to its agent-based variant by Judd (1998). Judd (1998, p. 13) takes a more or less defensive position when arguing that numerical results might serve as a solution good enough for decision-makers that are likely not to be interested in the proof of a theorem:

> *'Most end-users will agree that the patterns produced by such computations are likely to represent general truths and tendencies, and so form a reasonable guide until a conclusive theorem comes along.'*

The reference to a theorem shows that Judd interprets computational results as an approximation to the deduction of a theorem, in line with Koopman's (Koopmans, 1957) interpretation of neoclassical economics as an axiomatic theory. Axelrod and Tesfatsion (2006, p. 1650) expand this theme when it concerns agent-based computational economics:

> *'Simulation in general, and ABM in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments.'*

Agent-based modelling also enables the modeller to address explicitly the position of complex systems relative to individual-based approaches, based on the aggregation of and the interactions between agents. As Epstein and Axtell (1996, p.16) write:

> *'Our point of departure in agent-based modelling is the individual: We give agents rules of behaviour and then spin the system forward in time and see what macroscopic social structures emerge. This approach contrasts sharply with the high aggregate perspective of macroeconomics, sociology, and certain subfields of political science, in which social aggregates like classes and states are posited ab initio. To that extend our work can be accurately characterised as "methodological individualist." However, we depart company with certain members of the individualist camp insofar as we believe that the collective structures, or "institutions," that emerge can have feedback effects in the agent population, altering the behaviour of individuals.*

> *Agent-based modelling allows us to study the interactions between individuals and institutions.'*

Different interpretations of the use of computer models in social sciences and policy applications exist. According to one interpretation, a computer model is considered primarily the implementation of an algorithm for numerically solving a model represented by mathematical equations. According to another interpretation, agent-based modelling is different from building mathematical models (Gilbert and Troitzsch, 2005). Following the position taken by Epstein (2006) this distinction will be challenged in the next chapter, in the sense that there does not exist a computer model that cannot be represented by mathematical equations as well.

2.7 Conclusions

In this chapter, a general overview is presented of the two main methodological approaches applied in the social sciences. The first is the systems approach, the second the individual-based approach. Both are introduced after highlighting the contrast between the natural and the social sciences. The possibility for a researcher to choose a specific methodology when building a model in the social sciences is identified with the absence of general laws. This absence especially has implications for the deductive nature of model building in general. Deductive models in the natural and engineering sciences most frequently can be considered restatements of fundamental conservation laws, for example, the conservation of mass of energy. In an applied model these laws themselves are not verified. As a result, the axiomatic nature of the fundamental laws use can be accepted universally by the researchers in the discipline.

Deductive models in the social sciences rely on axioms as explicit assumptions or postulates. As a consequence, the use of these models always consists of a thought experiment. In this respect, microeconomics adopts an individual-based approach, based on certain postulates. Although these postulates are problematic in several cases, in this thesis the formulation of postulates at the level of individuals is preferred over assumptions regarding the behaviour at the level of aggregates or systems. A systems approach frequently relies on mathematical control theory. From a methodological point of view, this can often be interpreted as a teleological perspective on the behaviour of the system, as if the stationary equilibrium of the system would be reached purposefully by the system. Nevertheless, the mathematics of complex dynamical systems can offer the possibility to assess self-organisation at the system level, if the behaviour of the aggregate can be related to the behaviour of interacting individuals. Examples of this approach can be found in mathematical biology, especially in evolutionary game theory (EGT).

Finally, an assessment of the behaviour of complex dynamical systems usually requires the use of numerical methods, due to non-linearities. Given the preferred interpretation of the system's behaviour as the result of the interactions of individuals, the computational method chosen could make the behaviour of individuals explicit. In this respect, the relation between simulation and computation might be closer than is sometimes claimed in the literature on the use of simulation in the social sciences. An interesting approach is suggested in the discipline of agent-based computational economics (ACE), where some researchers propose to replace a mathematical proof by a credible simulation run.

Chapter 3

Complexity and evolutionary game theory

3.1 Introduction

In the relevant literature, there is no easily distinguished comprehensive ‘complexity theory’. If one defines a *complexity science*, it would deal with elements from other theories, especially from mathematics and computer science. Examples of applications can be found in physics, biology and disciplines that use mathematics extensively in modelling exercises. Complexity in mathematics most often refers to dynamical systems and is used there as a collective term for phenomena that occur in models with certain characteristics, covered more specifically in sub-branches such as catastrophe theory and chaos theory (Arnold et al., 1994; Strogatz, 1996) for example. This type of complexity will hereafter be referred to as *complex dynamical systems*. Section 3.2 will be devoted to it. Two characteristics of complex systems frequently referred to are *emergence* and *self-organisation*. As discussed in chapter 2, both characteristics can be identified with the occurrence of structure and the formation of patterns at an aggregate level. They have their origin however, in simple rules at the level of individual elements.

In section 3.3 it will be shown how mathematical biology—and especially *evolutionary game theory*—inspired economists in linking concepts from individual-based approaches to concepts from complex dynamical systems. In chapter 2, the relation between individuals and systems in mathematical biology was stressed. In this chapter, the focus is rather on the rules from which an elementary notion of *coordination* can be shown to emerge. The key concept at the individual level that governs coordination as an evolutionary process level is the so-called *best response*, which will be discussed in section 3.4. Especially relevant for agent-based modelling is the theory that deals with *computational complexity* that will be discussed in section 3.5. Computational complexity addresses the theoretical results about the possibility of—and time needed for—solving mathematical problems mechanically. A brief introduction will be presented on how both complex dynamical systems and computational complexity can be positioned relative to the neoclassical general equilibrium framework of microeconomics in section 3.6. Finally, in section 3.7 the conclusions of this chapter will be summarised.

3.2 Complex dynamical systems

The main characteristics of complex dynamical systems at the aggregate level are *non-linearity* and *interdependence*. Many models that can be considered complex consist of sets of ordinary differential equations. However, related sets of *partial* differential equations, ordinary and partial *difference* equations, as well as particle systems can be shown to possess similar notions of complexity. Self-organisation in a complex dynamical system can primarily be identified with the presence of a *feed-*

back mechanism, as formalised in control theory. If a variable in the model denotes the *state* of the system, a feedback mechanism uses this state variable again as input for determining the new state the next time step. For example, a thermostat uses the current temperature as input for keeping the temperature at a required level. If input of the state variables follows after a nonlinear transformation, a nonlinear feedback mechanism is present in the system. Finally, the sign of the input has a major influence on the behaviour of the system. In general, a negative feedback stabilises the system while a positive feedback destabilises it.

The same caveat that applies to systems theories—as discussed in section 2.4 —applies to 'complex dynamical systems'. Theories on complex dynamical systems cannot be isolated from the discipline in which they are applied. It is not difficult to find an engineering 'artifact' that complies with the formalisation of control theory. A popular example is a mass-spring oscillator with damping (Andronov et al., 1966). Some forced mass-spring oscillators can be shown to exhibit truly 'chaotic' behaviour and would thereby comply with the characteristics of a 'complex system', in the strict sense of the mathematical theory on dynamical systems. The position of the mass, the mass itself, gravity, and the interplay between the velocity of the driving force and the velocity and acceleration of the mass together allow for the formulation of a mathematical model that accurately explains the seemingly random—but actually chaotic—orbits of the mass. The 'complexity' that accounts for the behaviour of the forced mass-spring oscillator still conforms to Newton's Laws of Motion.

3.2.1 Canonical example of chaos

Perhaps the most popular example of complex behaviour is represented by the discrete version of the logistic growth model (adapted from May, 1976). The development of the size of a population, y, might be stated as the relative growth. This relative growth might—in turn—depend only on a net *growth rate* τ:

$$\frac{\dot{y}}{y} = \tau \tag{3.2.1}$$

The net growth rate can be thought of as the sum of a birth and death rate respectively:

$$\tau = \beta - \delta \tag{3.2.2}$$

If both β and δ are constants, the model is linear, since the equation can alternatively be written as $\dot{y} = (\beta - \delta)\, y$ and the y enters as a linear term on the right-hand side of the equation. A non-linear term can be introduced by making one rate or both rates, dependent on the current population size. A well-known example contains a death rate defined as

$$\delta\left(y\right)=\beta\frac{y}{K}. \tag{3.2.3}$$

With this definition of the death rate, the population size follows a *logistic growth*:

$$\dot{y}=\beta y\left(1-\frac{y}{K}\right). \tag{3.2.4}$$

The logistic growth model is a simple non-linear model. It reaches an equilibrium population size if $\dot{y}=0$. If the initial population is larger than zero, the equilibrium population size is given by $y=K$, where K is the carrying capacity. Written as a difference instead of a differential equation, the model can already show real chaotic behaviour for a certain parameter value[1]. The difference equation can be written as

$$\frac{y_{t+1}-y_t}{\Delta t}=\beta y_t\left(1-\frac{y_t}{K}\right). \tag{3.2.5}$$

With $\beta'=1+\beta\Delta t$ and $K'=K(1+\beta\Delta t)/\beta\Delta t$, this expression can be rewritten again as

$$y_{t+1}=\beta' y_t\left(1-\frac{y_t}{K'}\right). \tag{3.2.6}$$

For $K'=1$ and $\beta'=4$, chaotic orbits emerge as is shown in figure 3.1. This example illustrates how a relatively simple non-linear feedback mechanism can result in complex behaviour. For later reference, it is interesting to note that it is actually a *negative* feedback that induces chaotic behaviour instead of stability. The chaotic orbit is in fact a special variant of an ‘equilibrium’ solution, although it is not a fixed point. In chaos theory, the chaotic orbit is called a *strange attractor*.

3.3 Evolutionary game theory

Mathematical models in theoretical biology offer a source of inspiration for other disciplines, including economics. One interesting aspect of mathematical biology is the way it deals with the relation between individuals and systems. Models of ecosystems refer implicitly or explicitly to the behaviour of individual animals and plants, while the result of the interaction between these individuals can be represented at the system level. A well-known example is the Lotka-Volterra system, or predator-prey model[2]. In mathematical terms, the system is directly related to the logistic growth

[1] Incidentally, the difference equation that corresponds to a difference equation (3.2.4) can also be interpreted as an elementary algorithm for numerical integration (Press et al., 2002).

[2] For a rigorous discussion of this model see May (1974).

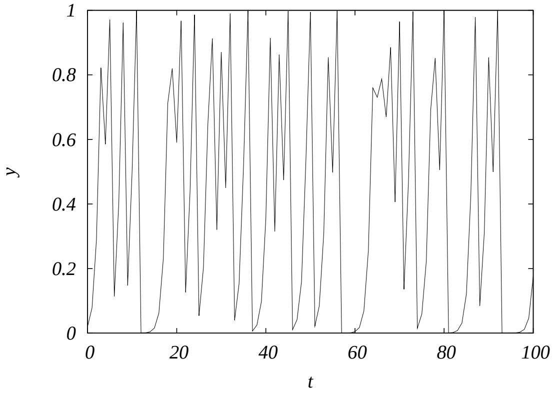

Figure 3.1: *Chaos in a discrete logistic growth model.*

model just mentioned. A prey population of size x is assumed to have a constant birth rate, α. Unlike in the logistic growth model, the death rate does not depend on the size of the own population, but on the size of predator population, y, multiplied by a coefficient β:

$$\frac{\dot{x}}{x} = \alpha - \beta y. \tag{3.3.1}$$

On the other hand, the predator population is assumed to have a growth rate that depends on the size of the prey population. Since no natural enemies exist for the predator population, the death rate is assumed to be constant. The dynamics of the predator population can therefore be represented by

$$\frac{\dot{y}}{y} = \gamma x - \delta. \tag{3.3.2}$$

Solving this systems results in an oscillating behaviour of both population sizes.

From a conceptual point of view, the interpretation at the individual level of the Lotka-Volterra model is limited. The population size is represented by a scalar value, while the dynamics are assumed to be continuous. Furthermore, birth and death rates are not suitable for a behavioural interpretation in terms of *decisions* made by individuals. A related model is the *replicator dynamics* from evolutionary game theory (Hofbauer and Sigmund, 1988; Weibull, 1995; Hofbauer and Sigmund, 1998; Vega-Redondo, 2003). This model explores the *strategic interaction* between populations of the same species. A population of fixed size N is divided into two subpopulations, n_1 and n_2, $(n_1 + n_2 = N)$. Instead of defining a growth rate for the population size, a growth rate for the fraction, $x_i = n_i/N$ with $i = 1, 2$, is defined according to

$$\frac{\dot{x}_i}{x_i} = f_i - \bar{f}. \tag{3.3.3}$$

Here, f_i denotes the *fitness* of the subpopulation i and $\bar{f}$ is the *mean fitness* with $\bar{f} = x_1 f_1 + x_2 f_2$. The fitness is defined similarly to the birth and death rates in the Lotka-Volterra model, since it depends on the current sizes of sub-populations:

$$f_i = a_{ii} x_i + a_{ij} x_j. \tag{3.3.4}$$

The replicator dynamics can for this two-subpopulations model can be represented by

$$\dot{x}_i = x_i \left[(A\mathbf{x})_i - \mathbf{x} \cdot A\mathbf{x} \right], \tag{3.3.5}$$

with

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \tag{3.3.6}$$

This matrix plays a central role in the next section. It is equivalent to a pay-off matrix in game theory. It enables the identification of fitness with utility and the replicator dynamics can be interpreted in terms of a *best response* in game theory, discussed in more detail in section 3.4. Similar to the logistic growth model, the values of $\mathbf{x}$ for which $\dot{\mathbf{x}} = \underline{0}$, or $f_i = \mathbf{x} \cdot A\mathbf{x}$ are of special interest. In classical game theory, a reference is usually made to two players. Each player is assumed to choose a strategy that is a best response to the strategy chosen by the opponent. Evolutionary game theory most frequently deals with a population of agents. The individual agents are assumed to choose a strategy that is a best response to the strategy all other individuals in the population choose. Instead of a rational choice, however, a process of natural selection is assumed. Natural selection here means that a strategy that proves not to be a best response to the choices of the population will not survive. In an evolutionary context, the question of whether a strategy will survive is considered subordinate to the question of whether individuals playing a certain strategy can *invade* an existing population. The connotation of an invasion in genetics originates in the concept of a *mutation*. An *Evolutionary Stable Strategy* (ESS) can be interpreted as a best response at the population level to which a ‘mutant’ playing a different strategy has no chance of surviving. Following Weibull (1995, p. 36), the condition for a mixed strategy, $\mathbf{x}$, to be an ESS can be formalised as

$$u\left[\mathbf{x}, \epsilon\mathbf{y} + (1 - \epsilon)\,\mathbf{x}\right] > u\left[\mathbf{y}, \epsilon\mathbf{y} + (1 - \epsilon)\,\mathbf{x}\right], \tag{3.3.7}$$

with ϵ sufficiently small. Here, $u(\mathbf{x}, \mathbf{y}) = \mathbf{x}A\mathbf{y}$, with A the payoff matrix as in (3.3.6). The condition (3.3.7) states that a population playing strategy $\mathbf{x}$ is resistant against a small group playing $\mathbf{y}$, since the fitness is still higher than if the whole population played $\mathbf{y}$. Also following Weibull (1995, p. 36), it can be concluded that an ESS is also an optimal strategy against itself. If there was a strategy $\mathbf{y}$ that would do better than $\mathbf{x}$ does against $\mathbf{x}$, it would in any event do better than a strategy $\mathbf{x}$ that is already 'flawed' by a fraction ϵ playing $\mathbf{y}$. In that case, according to (3.3.7), $\mathbf{x}$ would not be an ESS. Hence, an ESS is an optimal strategy against itself.

In the dynamics of (3.3.5) the notion of resistance is captured in the dynamic stability of the equilibrium solution $\dot{x}_j = 0$. It can be thought of as a possible example of a dynamics that enforces the strict inequality, $>$, in (3.3.7). Once in equilibrium, any mutant strategy will yield a lower fitness than the average and the mutant fraction will decline over time. Its dynamics are analogous to that of a pendulum for example. After any perturbation, ϵ, that is small enough it will return to its equilibrium state. Therefore, the replicator dynamics can be interpreted as a system with a stabilising negative feedback mechanism. Furthermore, it conforms to a basic notion of *self-organisation*, as the equilibrium results from a process that involves interacting individuals. With respect to the two-agent model developed in chapter 4 and the population model in chapter 6, the ESS is especially interesting, because it is a *refinement* of the Nash equilibrium concept of classical game theory that is discussed in the next section. It therefore also allows for a normative interpretation of the equilibrium solution.

3.4 Best response and multiple equilibria

The concept of the *Nash equilibrium* is central to the normative interpretation of how strategic interaction between agents in game theory[3] is formalised. A Nash equilibrium can be defined more or less informally as a 'best response to a best response'. Although the classical interpretation requires rather unrealistic cognitive capacities from the agents, concepts from evolutionary game theory and learning algorithms in computer science allow for bounded rational interpretation. In the light of complexity, the concept of the Nash equilibrium is interesting because a game might have multiple Nash equilibria. A dynamical system with multiple equilibria can be considered a more 'advanced' complex system. If the equilibrium of a deterministic dynamical system is unique, it will always mark the end point of a process regardless of the initial conditions. Multiple equilibra potentially introduce *path dependency*. If the number of equilibria is dependent on a parameter, *bifurcations* occur for every value of this parameter at which the number of equilibria changes.

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[3] The references made to game theory in this thesis all concern *non-cooperative* games.

First, the concept of a best response will be formalised in a simple game[4]. A game played by two agents is defined by the strategies and the payoffs agents receive when playing the strategies. The payoffs can be represented by two matrices. For agent 1 the payoff matrix is given by

		Agent 2 Strategy I	 Strategy II
Agent 1	Strategy I	(a_{11}, b_{11})	(a_{12}, b_{21})
	Strategy II	(a_{21}, b_{12})	(a_{22}, b_{22})

Table 3.1: *Two-agent game in normal form.*

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{3.4.1}$$

Based on this matrix, an expected payoff—or expected utility[5]—can be defined as a function of the *probability* that the opponent plays one of his strategies. With x_2 as the probability that agent 2 plays his first strategy, the expected payoffs for the respective strategies of agent 1 are given by

$$v_{11} = x_2 a_{11} + (1 - x_2)\, a_{12} \tag{3.4.2a}$$

$$v_{12} = x_2 a_{21} + (1 - x_2)\, a_{22}. \tag{3.4.2b}$$

The decision of which strategy to choose can be expressed by means of a *rule*, as

$$\begin{array}{llll} \text{if} & (v_{11} > v_{12}) & \text{then} & x_1 = 1, \\ \text{if} & (v_{11} \leqslant v_{12}) & \text{then} & x_1 = 0. \end{array} \tag{3.4.3}$$

In case the expected utility of strategy *I* is higher than that of strategy *II*, the agent is supposed to opt for strategy *I*.

The decision rule (3.4.3) is basically a *best response* to the belief agent 1 has, concerning the strategy agent 2 will play. This belief is expressed as the probability x_2. Agent 1 can also decide to play both strategies with a certain probability x_1 for playing strategy *I*. To determine the best response to x_2, a rational agent would first calculate the value of x_2 for which $v_{11} = v_{12}$. If agent 2 plays exactly this strategy,

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[4] In this thesis, only games represented in the so-called *normal form* will be referred to. This representation corresponds in classical (non-evolutionary) game theory to simultaneous moves by the agents playing the game.

[5] The concept of expected utility was introduced by von Neumann and Morgenstern (1944).

agent 1 would be indifferent between playing strategy *I* or strategy *II*. This value can easily be calculated from (3.4.2) and yields

$$\hat{x}_2 = \frac{(a_{12} - a_{22})}{(a_{12} - a_{22}) - (a_{11} - a_{21})}. \tag{3.4.4}$$

From (3.4.4) it can be learnt that only the differences between certain parameter pairs are important in determining the best response. This allows for a simplification of the payoff matrix A. With $a_1 \equiv a_{11} - a_{21}$ and $a_2 \equiv a_{22} - a_{12}$ the matrix can be represented by

$$\widetilde{A} = \left[\begin{array}{cc} a_1 & 0 \\ 0 & a_2 \end{array} \right]. \tag{3.4.5}$$

The matrix $\widetilde{A}$ in combination with (3.4.2) can be used for the identification of four types of best responses for a 2 by 2 payoff matrix. Using (3.4.5), the value for which the agent is indifferent between playing strategy *I* or *II*, (3.4.4) can be rewritten as

$$\hat{x}_2 = \frac{a_2}{a_1 + a_2}. \tag{3.4.6}$$

Depending on the signs of a_1 and a_2, it can be determined whether $0 < \hat{x}_2 < 1$ and whether $v_{11} > v_{12}$ in case $x_2 \neq \hat{x}_2$. The four types of best responses are presented in in table 3.2.

1	$a_1 > 0$	$a_2 > 0$	$x_1 = 0 \quad \text{if} \quad x_2 < \hat{x}_2$	$x_1 = 1 \quad \text{if} \quad x_2 > \hat{x}_2$
2	$a_1 > 0$	$a_2 < 0$	$x_1 = 1$	
3	$a_1 < 0$	$a_2 > 0$	$x_1 = 0$	
4	$a_1 < 0$	$a_2 < 0$	$x_1 = 1 \quad \text{if} \quad x_2 < \hat{x}_2$	$x_1 = 0 \quad \text{if} \quad x_2 > \hat{x}_2$

Table 3.2: *Four types of best response.*

3.4.1 Coordination and lock-in

The first case is of special interest. For the game with the following payoff matrix:

$$\widetilde{A} = \left[\begin{array}{cc} 3 & 0 \\ 0 & 2 \end{array} \right], \tag{3.4.7}$$

the corresponding best response function that reflects the rule of (3.4.3), is plotted in figure 3.2 with $\hat{x}_2 = 0.4$. Agent 1 is supposed to play strategy *II* in case $x_2 < \hat{x}_2$ and strategy *I* if $x_2 > \hat{x}_2$. If it assumed that the agents are identical, the payoff matrix for the other agent, B, would be identical to the payoff matrix of the first agent: $B = A$. This denotes a *symmetrical* game. Nash equilibria can be represented graphically by plotting the best responses for both agents in one figure. This is done in figure 3.3. Using (3.4.6), and with $x = x_1 = x_2$, the Nash equilibria are found to be $x = 0$, $x = 0.4$ and $x = 1$. The relation with the replicator dynamics can now be introduced as follows. According to the definitions, the fitness in the replicator dynamics is identical to the expected utility in classical game theory. The replicator dynamics introduces a notion of dynamic stability to the game theoretical interpretation. Because

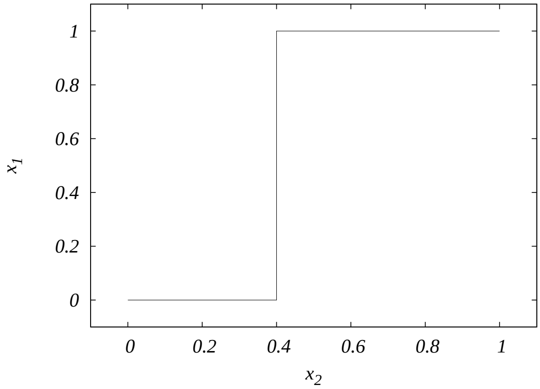

Figure 3.2: *Best response for* (3.4.7).

$$\begin{aligned} f_1 - \overline{f} &= f_1 - x f_1 - (1 - x) f_2 \\ &= (1 - x)(f_1 - f_2), \end{aligned} \tag{3.4.8}$$

it follows that

$$\dot{x} = x(1 - x)(f_1 - f_2) = 0, \tag{3.4.9}$$

has three solutions. It is to be noted that for $x \in (0, 1,)$, both x and $(1 - x)$ on the right-hand side have a positive value. The sign of the disequilibrium dynamics is therefore determined by $f_1 - f_2$. The dynamics will evolve in the direction in favour of strategy 1 if $f_1 > f_2$ and in favour of strategy s if $f_1 < f_2$. In section 3.3, it was proven that an ESS is a best strategy against itself. It can therefore be concluded that every ESS is also a Nash equilibrium, since an ESS is a best response to a best

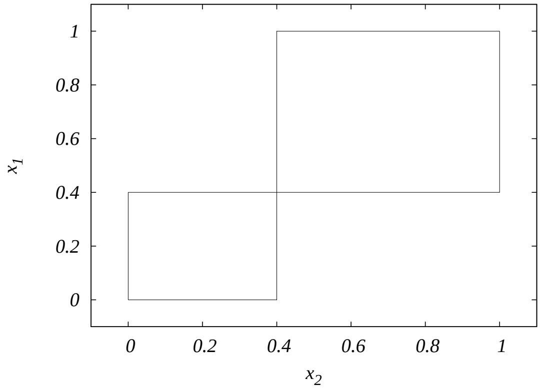

Figure 3.3: *Nash equilibrium (best response to best response).*

response in a symmetric game. However, not every Nash equilibrium is an ESS, as can be seen for $x = 0.4$ in the example above. Although, this strategy mix is a Nash equilibrium, it is not evolutionary stable. A mutation, ϵ, would enforce a situation in which $f_1 \neq f_2$, and the dynamics would evolve in favour of one of the two strategies, depending on the sign of ϵ.

A game with a payoff matrix of type 1 in table 3.2 for both agents is an example of a *coordination game*. With respect to complexity, the evolutionary game theoretical interpretation of the coordination game is of special interest because the two Nash equilibria in pure strategies are evolutionary stable, whereas the Nash equilibrium in mixed strategies is not. At the level of the population, this implies that either the entire population plays strategy *I* or strategy *II*. With a deterministic evolutionary development as reflected in the replicator dynamics, the starting point determines which solution is the equilibrium solution. Since both evolutionary stable strategies are Nash equilibria, both solutions are optimal for the *individual* agent with respect to the strategies of the other agents. However, the payoff matrix in (3.4.7) serves as an illustration of the possibility that one solution can be better than the other at the *aggregate* level. In (3.4.7), strategy *I* is said to be *Pareto dominant* to strategy *II*. If the population plays strategy *II*, everyone could in theory be made better off without making anyone worse-off, if all agents would switch collectively to strategy *I*. In terms of evolutionary economics, strategy *II* would represent a *lock-in* at the level of a society. Especially in the context of an evolutionary selection mechanism that has possibly lead to this equilibrium, this interpretation might be considered a powerful metaphor. It integrates insights from complex dynamical systems with a traditional normative interpretation.

Finally, with respect to a *bifurcation* from one equilibrium to multiple equilibria, the following variant of (3.4.5), with $a = a_1 = a_2$, is important:

$$\tilde{A} = \left[\begin{array}{cc} a & 0 \\ 0 & a \end{array} \right]. \tag{3.4.10}$$

Table 3.2 now reduces to table 3.3. The parameter a now serves as a *bifurcation* *parameter*. Depending on its sign, the system has either one or two ESS. In the context of a population game this interpretation is relatively abstract. In chapter 4 a similar construction will be discussed concerning network externalities.

1	$a > 0$	$x = 0 \quad \text{if} \quad x < \hat{x}_2$	$x = 1 \quad \text{if} \quad x > \hat{x}$
2	$a < 0$	$x = \hat{x}$	

Table 3.3: *Two types of best response for* (3.4.10).

3.4.2 Smoothed best response

Primarily for the reference in chapter 4, the best response for an expected utility with an additional stochastic term will be introduced here. It is discussed in Fudenberg and Levine (1998, p. 101–133) in the context of *learning in games*. The main reason for referring to it in this chapter, is the possibility of defining a *continuous* function that can approximate the discrete choice between two options. A continuous best response function is helpful in making the translation from dynamical systems as systems of differential equations to individual agents in an agent-based model, and back. It also facilitates the interpretation of a best response in terms of *probabilities* in a stochastic environment.

Starting with (3.4.3) an error term will be added on both sides of the equation:

$$\begin{aligned} &\text{if} \quad (v_{11} + \mu\varepsilon_{11} > v_{12} + \mu\varepsilon_{12}) \quad \text{then} \quad x_1 = 1, \\ &\text{if} \quad (v_{11} + \mu\varepsilon_{11} \leqslant v_{12} + \mu\varepsilon_{12}) \quad \text{then} \quad x_1 = 0. \end{aligned} \tag{3.4.11}$$

Assuming that the difference between the error terms has a *logistic* distribution[6], the cumulative probability density function approximates the best response curve, as is illustrated in figure 3.4. This smoothed best response, x_1, can be expressed as

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[6] For a more detailed discussion on this issue see chapter 4.

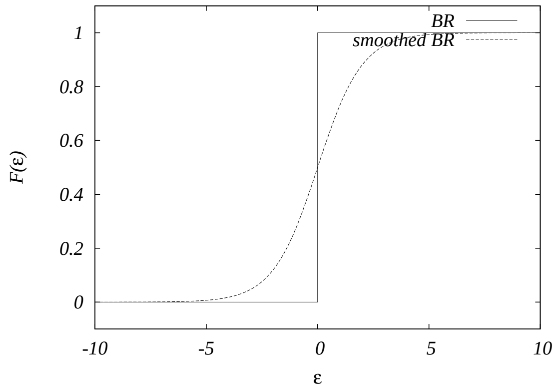

Figure 3.4: *Logistic distribution and best response.*

$$x_1 = \frac{\exp\left[\frac{v_{11}(x_2)-v_{12}(x_2)}{\mu}\right]}{\exp\left[\frac{v_{11}(x_2)-v_{12}(x_2)}{\mu}\right]+1}. \tag{3.4.12}$$

Using this definition, a Nash equilibrium can be approximated as a smoothed best response to a smoothed best response, as illustrated by figures 3.5 and 3.6. Due to the stochastic term, this best response actually concerns a *probability*. Therefore the term ‘best response *correspondence*’ is frequently used for this variant.

3.5 Computational Complexity

In computer science, complexity theory deals with *computability* and is also called *computational complexity* (Sedgewick and Flajolet, 1996). It refers to a classification of algorithms by their capability and speed for solving problems. The definition of ‘complexity’ in this context is different from its use in dynamical systems. Some overlap may occur because systems of non-linear equations frequently can only be solved using computer algorithms. However, algorithms for solving differential equations usually only involve numerical integration, while fundamental questions regarding computability are related to existence proofs. The use of the terminology derived from computational complexity in disciplines other than computer science is less common than the use of terminology from complex dynamical systems, with the exception of Operations Research (Simon, 1996).

Computational complexity classifies algorithms according to the length of time

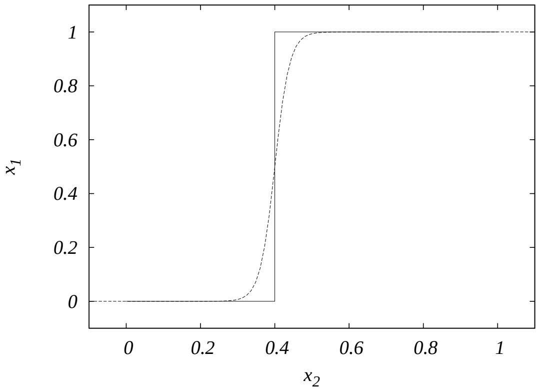

Figure 3.5: *Smoothed best response.*

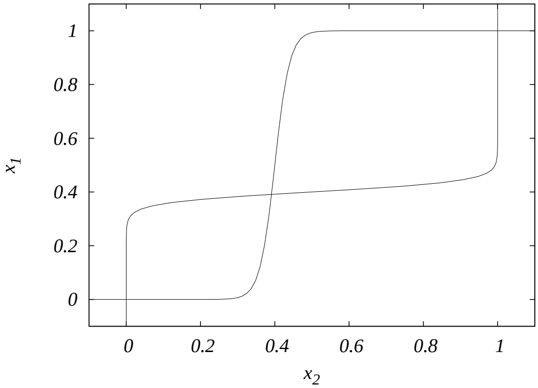

Figure 3.6: *Nash equilibrium in smoothed best responses.*

needed for accomplishing their tasks. Time can be expressed in the number of operations required. The theory deals primarily with the limits on the number of operations needed, as the size of the problem goes to infinity. Illustrative examples can be found in sorting algorithms in which the size is determined by the number of elements to be sorted. For sorting a list of book titles alphabetically, different approaches exist. A search algorithm that sorts by the method of ‘divide and conquer’—where the list is split first in two halves, the relevant half is split in two quarters of the original list etc.—is faster than a method that checks every single element, starting from the first.

Related to the complexity of the algorithm is the possibility to solve a given problem at all. If a problem can be solved, the complexity of the problem might be identified with the complexity of the algorithm that has the best performance. If it is not clear beforehand whether a given problem can be solved, a strict definition of what an algorithm does, is needed. Put differently, a definition of *computation* is required for the formalisation and generalisation of algorithms. This definition of computation, in turn, allows for a strict definition of the computability of a given problem. According to this theory if a problem cannot be computed, it cannot be solved.

The concept of computability originates in the fundamental research on the logical foundations of mathematics at the beginning of the 20^{th} century. Computation was defined as a task that might be performed by a limited set of operations and served as a formalisation of a deductive proof in mathematics. The computation was thought to be implemented mechanically in the theoretical concept of a *Turing machine*. Implementation in real machines has eventually led to the development of the modern computer. The so-called *assembly language* which allows programmers to write programmes almost directly in machine code is essentially still based on the three basic operations Turing proposed for the universal variant of his machine. Since machine code defines the operations at the lowest level of any computer, Turing’s definition of computation is relevant for any type of computer model, including agent-based models.

The original scientific ambition of expressing all mathematical propositions in a computable fashion has failed. Gödel’s famous Incompleteness Theorems showed that in every axiomatic theory it will be possible to express propositions that cannot be proven in the mechanical way described above. The first theorem states that no formal theory is *complete*, meaning that it is always possible to formulate a statement that is true, but cannot be proven within the theory. The proof of this theorem relies on computability, in the sense that attempts to prove the true statement fails because they all result in a contradiction. The equivalence to a contradiction in computation is an algorithm that does not stop. Gödel’s second theorem states that every formal theory contains a statement of this type of contradiction, claiming that the theory is consistent. Since it is a contradiction, the theory itself is inconsistent if and only if it contains a statement that claims its own consistency. The canonical type of this contradiction is similar to a paradox in ordinary language, such as the story of the Cretan (Epimenides) who claimed that all Cretans are liars.

The tools developed for defining *computability* highlight the similarity with other deductive systems that use equations. If individuals are represented as *objects* in computer code, a model description will concern an algorithm rather than a set of equations. The degree to which a group of autonomous agents can solve a problem successfully as a collective is still bound by the limits imposed by the hardware of computers. Since theoretical results on computability inspired the construction of computers, behavioural theories implemented in an agent-based model are directly related to theories on artificial intelligence and algorithms. It was argued in section 2.6, with a reference to Epstein (2006), that agent-based models are not fundamentally distinct from models represented by sets of equations. First of all it has to be noted that the type of equations Epstein refers to is rather specific. Given this special type, the argument runs as follows:

- Every Turing Machine can be represented by a *partial recursive function*,
- Every computer operates as a *universal*[7] Turing Machine,
- Hence, every computer model can be represented by a system of partial recursive functions.

The term *recursion* refers—more or less informally—to the composition of an object by elements of the same type as the object itself. If presented visually, it is sometimes—and especially in The Netherlands—referred to as the *Droste effect*, after the image of a nurse on a box of cacao from the Dutch brand ‘Droste’. The nurse holds in her hands a box, with an image of nurse, holding in her hands..., etc. Another example is the name of a part of the *open source movement* in software development: GNU. It is an abbreviation for *GNU is Not Unix*; defining the meaning of the letter G recursively. A recursive function in mathematics and computer science is strictly defined only in terms of a Turing Machine and computability. Therefore a short overview of the concept of a Turing Machine is presented next.

3.5.1 Turing Machine

The description presented here is adapted from Crossley et al. (1972, p. 31–44). A Turing Machine should be thought of as a *reading head* that scans a tape. The tape consists of squares with symbols. Some squares can be empty. In the theory, the tape

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[7] See footnote 8.

can be infinitely long. The reading head is capable of performing the following three tasks:

1. change the scanned symbol,
2. move the tape one square to the left,
3. move the tape one square to the right.

The task the reading head performs depends on its *state*. After it has performed a task, its state might change. A basic instruction for the machine can therefore be represented by *four* variables:

1. the current state,
2. the current symbol on the scanned square,
3. the changed symbol, or the direction of moving the tape on square (L or R),
4. the next state.

The reading head recognises instructions by combinations of the first two symbols: current state and current symbol. If there is no instruction that matches the current combination of state and symbol, the machine stops. If the state is denoted by q_i, an empty square by $\square$ and if all symbols are based on the number 1, possibly altered by a ' , all instructions can be composed by an alphabet consisting of only six symbols: q, $\square$, 1, ' , L and R.

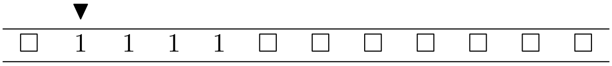
Now the definition can be stated: A function $\phi(n)$ is a *partial recursive function* if there exists a Turing Machine M that starts with a number of n symbols '1' on the right-hand side of the reading head and stops with a number of $\phi(n)$ symbols. Because the Turing Machine might not be defined for every n, the adjective *partial* is added to this definition of a *recursive function*. As an example of a partial recursive function and the associated Turing Machine, Crossley et al. (1972, p. 36) present the function $\phi(n) = 2n$. For any given n, this function should basically copy all symbols. It is assumed that when the machine starts, the reading head is positioned above the empty square, left to the first '1' of the set to be copied:

▼											
□	1	1	1	1	□	□	□	□	□	□	□

In order to let the 'programme' start, the first instruction is

$$(q_0, \square, R, q_0)\,. \tag{3.5.1}$$

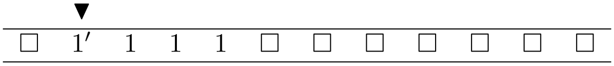
This instruction positions the reading head just above first square with a '1' on it:



The second instruction is

$$(q_0, 1, 1', q_1)\,. \tag{3.5.2}$$

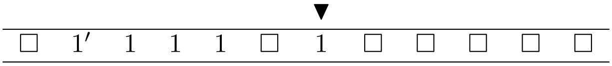
The reading head changes the first ‘1’ in a ‘$1'$’:



It then searches for an empty square on the right-hand side of the ‘$1'$’ that was just created. If it finds an empty square, it skips one square and writes a new ‘1’ on the next empty square. After that, the state is switched to q_3. The instructions are

$$\begin{aligned}&(q_1, 1', R, q_1)\,,\\&(q_1, 1, R, q_1)\,,\\&(q_1, \Box, R, q_2)\,,\\&(q_2, \Box, 1, q_3)\,.\end{aligned} \tag{3.5.3}$$

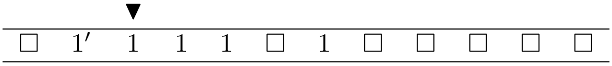
The result on the tape is



After that it goes back, searching for the ‘$1'$’ on the left-hand side. It moves one additional square to the right again in case it has found it, and switches to state q_4:

$$\begin{aligned}&(q_3, 1, L, q_3)\,,\\&(q_3, \Box, L, q_3)\,,\\&(q_3, 1', R, q_4)\,.\end{aligned} \tag{3.5.4}$$

On the tape, the result is



From state q_4 it can switch to q_0 again for copying the next ‘1’:

$$(q_4, 1, 1, q_0)\,. \tag{3.5.5}$$

Finally, an instruction is needed for finishing the procedure. The procedure should stop if all n symbols of the original set are marked. This is the case if the machine is

1	q_0	□	R	q_0
2	q_0	1	$1'$	q_1
3	q_1	□	R	q_2
4	q_1	1	R	q_1
5	q_1	$1'$	R	q_1
6	q_2	□	1	q_3
7	q_3	□	L	q_3
8	q_3	1	L	q_3
9	q_3	$1'$	R	q_4
10	q_4	□	L	q_5
11	q_5	1	L	q_5
12	q_5	$1'$	1	q_5

Table 3.4: *Instructions needed for representing* $\phi(n) = 2n$.

in state q_4, but the reading head is above the empty square that was left between the original set and its copy. If it is above the empty square, it moves one square to the left; the last symbol of the original set:

$$(q_4, \Box, L, q_5)\,. \tag{3.5.6}$$

If in state q_5, all marks on the symbols of the original set should be removed:

$$\begin{aligned}&(q_5, 1', 1, q_5)\,,\\ &(q_5, 1, L, q_5)\,.\end{aligned} \tag{3.5.7}$$

This procedure continues until the reading head is positioned left of the first symbol of the original set, since it has no instruction that matches the combination q_5 and $\Box$. Left to the reading head, there is now a number of $2n$ symbols ‘1’.

▼											
□	1	1	1	1	□	1	1	1	1	□	□

The relevance of this example of a Turing Machine with regards to the discussion on complexity lies in the observation that the function $\phi(n) = 2n$ can be represented by a *mechanical procedure* that employs a *finite* number of instructions. In this case 12 instructions are sufficient. They are summarised in table 3.4. The instructions in turn, are constructed with a finite number of elements; the *alphabet* as it was referred to above. In principle, the finiteness of both instructions and alphabet allows in principle for a translation of all components of a Turing Machine to new alphabet—or coding system—that consists only of ‘1”s and ‘0”s. The construction of the central processing unit (CPU) of a digital computer is based on this principle, with *machine code* as the most elemental language, translated from assembly language.

Even if an agent-based model is coded using an object-oriented language, as discussed in section 2.6, a *compiler* translates the programming code of the language to machine code. Agent-based models are therefore eventually processed by a CPU at the level of basic instructions inspired by a Turing Machine[8]. Because a partial recursive function can be represented by a Turing Machine, an agent-based model might alternatively be represented by a system of partial recursive functions. This system will probably be less illuminating than the organisation of the behavioural rules in a computer programme written in an object-oriented language. Nevertheless, the equivalence implies that there is no fundamental distinction between an agent-based model and a model expressed in mathematical equations.

3.6 Neoclassical Economics and complexity

After the overview of complexity issues in general, an short introduction on the role insights from complex dynamical systems and computational complexity might play in an assessment of neoclassical economics is given. First of all, the linearity in the neoclassical framework refers to *linear spaces*, rather than linear equations (Koopmans, 1957). As before, ‘neoclassical economics’ here refers to the Walrasian general equilibrium framework in the formalisation by Arrow and Debreu (1954) (see also Debreu, 1959). The main benefit a researcher gains from a linear activity model is the possibility to *decompose* the behaviour of an aggregate model into the separate contributions of the elements. Together, these contributions result in the behaviour of the aggregate system, simply by means of *addition*. The linear spaces in the neoclassical general equilibrium model concern the *commodity sets* for consumers and producers. If the commodity sets for two consumers are defined within a linear space, the sum of the commodity bundles chosen by the consumers represents the total amount of commodities chosen. Rationality implies that a consumer chooses a *preferred* bundles from the set of possible bundles. If the preferred bundles of the agents are *independent* of each other, the basic condition for the First Welfare Theorem is already fulfilled. This follows from the possibility for each consumer to maximise his utility by choosing the most preferred bundle. As a result, the total demand for the commodities will contain the optimal demand for each individual agent. It immediately follows that if the preference structure of one agent depends on the decision another agent makes, total demand following independent

[8] A computer is actually a ‘Universal Turing Machine’. This is a conceptual generalisation, based on the idea that the tape for a Universal Turing Machine could start with the basic set of instructions for some Turing Machine—for example, the instructions of table 3.4. This set enables the Universal Turing Machine to replicate the results of any Turing Machine.

utility maximisation will be different from total demand resulting from maximisation under interaction. Results from the mathematical theories on non-linear systems therefore have a clear interpretation in economics. Non-linearity arises if *non-market interactions* are added to a model of a market equilibrium. Non-market interactions can therefore be considered an example of extending the neoclassical general equilibrium model with elements from complexity theory.

As noted in chapter 1, parallel to the dominance of neoclassical economics in the 1950s and 1960s, *game theory* complemented the economist's tools for assessing a wider set of markets, oligopoly for example, by modelling strategic interactions. But in the 1990's, *evolutionary game theory* (Weibull, 1995) in particular gave rise to the development of approaches in which pure strategic considerations were replaced by the emergence of informal institutions, or conventions. A key feature of these approaches is the redefinition of strategic interaction as *social context*. It offers a natural way to complement, rather than replace, the Arrow-Debreu framework. Following the more recent literature on game theory (see for example Gintis, 2000), the concept of a game has a very general interpretation. Basically any situation in which different agents have the same or different interests and in which some form of dependency exists can be formalised as a game. A Nash equilibrium corresponds in that case to any formal or informal agreement. By definition of a Nash equilibrium this agreement reflects the best option for the individual agent, given the best options of all other agents. Combined with a 'short-sighted' variant of bounded rationality—or myopic utility improvement—, stylised versions of civilisations were constructed, in which people still act in their own interest, but together spontaneously establish certain rules of collective behaviour, such as driving on the right (or left) hand side of the road or eating with knife and fork (Young, 1998). These are examples of the evolutionary approaches to the coordination game discussed in section 3.4. Translated to the level of a society, this agreement can be a formal or an informal *institution* (Young, 1998). An example of formal institution is a *law*, while an example of an informal institution is usually related to traditional rules of conduct. The distinction is not always very strict. If a law is enforced on agents with democratic legitimacy, in principle consensus exists among all agents. Whether traffic drives on the left- or the right-hand side of the road, for example, depends on a country's history. If one side has become part of a national law, this still reflects the general consensus that the roads are most safe if everybody cooperates with this societal agreement. Reintroduction of price considerations in this social context established the field of the (New) Social Economics (Becker and Murphy, 2000; Durlauf and Young, 2001), where economists try to integrate concepts similar to those from mathematical sociology (Coleman, 1990) into an economic-theoretical analysis.

As discussed in section 3.4, the interpretation of a coordination game in terms of a population game offers a powerful metaphor of a more advanced type of a complex dynamical system due to the presence of multiple equilibria. But even in the case of a unique equilibrium and market interactions only, the notion of *self-organisation* can possibly contribute to an extension of the neoclassical framework. Adam Smith's *Invisible Hand* can be considered an example of self-organisation. However, the neoclassical framework contains only a formalisation of the idea that, if all information about a product is contained in its price, a price vector exists that supports an efficient allocation of goods. It does not describe the process in which the optimal price vector emerges. Chapter 4 is devoted to this discussion.

The concept of *computability* from computational complexity provides the possibility of extending the discussion on the selection of the neoclassical market equilibrium. Although a market equilibrium might be computed by an optimisation algorithm, the credibility of the existence of the optimal equilibrium in reality can be made dependent on the behavioural interpretation of this algorithm. A computer simulation model can assist in the formalisation of what is 'decentral' in a decentralised allocation. The agents acting in their self-interest—corresponding to Adam Smith's original interpretation of the Indivisible Hand—can be modelled as autonomous self-interested agents, as in Epstein and Axtell (1996). The possibility of replicating a neoclassical market equilibrium is then a benchmark for a special kind of computability with additional conditions. This discussion was introduced in section 2.6 as one of the research topics in agent-based computational economics, and will be extended in chapter 4.

3.7 Conclusions

Two types of complexity are discussed in this chapter: complex dynamical systems and computational complexity. The first is shown to be relatively general. It can be considered a branch of mathematics and control theory and problems arise with the interpretation as an independent theory. These problems are similar to those related to systems approaches in general, discussed in section 2.4. Mathematical biology provides not only examples of integrating individual- with systems-based approaches; it can also provide an interpretation of dependency and non-linearity with respect to individual behaviour. Especially evolutionary game theory offers a framework in which key characteristics of complex dynamical systems—such as self-organisation, multiple equilibria, bifurcations and path dependency—can be related to the more traditional normative concept of a Nash equilibrium.

Computational complexity provides a precise definition of computability. Its interpretation as a series of mechanical operations can be related directly to the hardware of computers. This interpretation plays an important role in the argument that computer models are not essentially different from deductive models represented by mathematical equations. It also offers suggestions for the identification of computability with a behavioural interpretation of agent-based models.

Both notions of complexity can be related to the neoclassical framework of microeconomics. The non-linearity of complex dynamical systems can be contrasted with the neoclassical linear activity model and used to define a basis for complementing neoclassical market interactions with non-market interactions. Even in the absence of non-market interactions, the notion of self-organisation seems to correspond to the process that is implicitly assumed to result in the optimal market equilibrium. Finally, this notion of self-organisation can be given an computational interpretation, using an agent-based algorithm indicated above.

3.8 Appendix

In this appendix derivations for both the discrete and continuous versions of the replicator dynamics are given. These derivations have their origin in biology, especially in genetics (Fisher, 1930). Alternative derivations that correspond to behavioural rules for agents will be presented in chapter 6.

3.8.1 Discrete

The derivation of the discrete version of the replicator dynamics presented below is based on Hofbauer and Sigmund (1988).

Given a population of size N, the fraction of the subspecies i is given by (with $\sum n_i = N$)

$$x_i\left(t\right) = \frac{n_i\left(t\right)}{N\left(t\right)}. \tag{3.8.1}$$

The size of next generation of the subspecies i, resulting from a meeting with subspecies j is given by

$$n_{ij}\left(t+1\right) = n_i\left(t\right) x_j\left(t\right) a_{ij}. \tag{3.8.2}$$

The probability x_j denotes the probability of a member of i meeting a member j and a_{ij} is the reproduction rate (in new members of i per current member of i). The total number of members of i in the next generation is therefore given by

$$n_i\left(t+1\right) = N\left(t\right) \sum_j x_i\left(t\right) x_j\left(t\right) a_{ij}. \tag{3.8.3}$$

The size of the total population follows from the summation over all sub-populations:

$$N(t+1) = \sum_i n_i(t+1) = N(t)\sum_i \sum_j x_i(t)\, x_j(t)\, a_{ij}. \tag{3.8.4}$$

Next, *fitness* is defined as the total reproduction rate per subspecies

$$f_i(t) = \sum_j x_j(t)\, a_{ij}. \tag{3.8.5}$$

The size of the new subspecies then reads

$$n_i(t+1) = n_i(t)\, f_i(t). \tag{3.8.6}$$

And the fraction in the new generation as

$$x_i(t+1) = \frac{n_i(t+1)}{N(t+1)} = \frac{x_i(t)\, f_i(t)}{\sum_j x_j(t)\, f_j(t)} = x_i(t)\,\frac{f_i(t)}{\overline{f}(t)}. \tag{3.8.7}$$

This expression maintains that the relative growth of fraction in the new generation depends on the fitness (reproduction rate) of the previous generation, relative to the average fitness level.

3.8.2 Continuous

The continuous case is adapted from Yazar (2006). With the definition of differential (instead of difference) equation

$$\dot{x}_i(t) = \lim_{\Delta\downarrow 0} \frac{x_i(t+\Delta) - x_i(t)}{\Delta}, \tag{3.8.8}$$

it is assumed that only the ‘fraction’ α of the fraction takes part in reproduction during interval Δ:

$$x_i(t+\Delta) = \frac{\alpha\Delta x_i(t)\, f_i(t) + (1-\alpha\Delta)\, x_i(t)}{\sum_j \alpha\Delta x_j(t)\, f_j(t) + (1-\alpha\Delta)\, x_j(t)}. \tag{3.8.9}$$

Note that the denominator can be rewritten as

$$\begin{aligned}
&\textstyle\sum_j \alpha\Delta x_j(t)\, f_j(t) + (1-\alpha\Delta)\, x_j(t) \\
&= \alpha\Delta \textstyle\sum_j x_j(t)\, f_j(t) + (1-\alpha\Delta)\textstyle\sum_j x_j(t) \\
&= \alpha\Delta\overline{f}(t) + (1-\alpha\Delta).
\end{aligned} \tag{3.8.10}$$

It follows that

$$\begin{aligned}\dot{x}_i\left(t\right) &= \lim_{\Delta\downarrow 0}\frac{1}{\Delta}\left\{\frac{\alpha\Delta x_i\left(t\right)f_i\left(t\right)+\left(1-\alpha\Delta\right)x_i\left(t\right)-x_i\left(t\right)\left[\alpha\Delta\overline{f}\left(t\right)+\left(1-\alpha\Delta\right)\right]}{\alpha\Delta\overline{f}\left(t\right)+\left(1-\alpha\Delta\right)}\right\}\\ &= \lim_{\Delta\downarrow 0}\frac{\alpha\Delta x_i\left(t\right)}{\Delta}\left\{\frac{f_i\left(t\right)-\overline{f}\left(t\right)}{\alpha\Delta\overline{f}\left(t\right)+\left(1-\alpha\Delta\right)}\right\}. \qquad (3.8.11)\end{aligned}$$

And finally

$$\dot{x}_i=\alpha x_i\left(f_i-\overline{f}\right). \qquad (3.8.12)$$

This is the replicator dynamics as introduced in section 3.3.

Chapter 4

Pareto efficiency and best response dynamics

4.1 Introduction

This chapter discusses the efficiency of a market allocation in the neoclassical framework, together with an evolutionary approach to the computation of the market equilibrium. An alternative interpretation of a neoclassical two-agent, two-commodity exchange economy based on the themes and elements collected in chapter 3 will also be developed.

Under well-specified conditions, in the neoclassical framework the equilibrium of demand and supply corresponds to Pareto efficient allocation. The framework, however, does not specify the process of price setting. A competitive market is characterised by price taking behaviour for all agents involved, which essentially implies that an outside institution determines the prices at which markets clear. In the literature, special attention is devoted to the search of a two person bargaining game that results in the same prices. The intuition behind this search is clear. Not in the least do policy interpretations of competitive markets often appeal to the *emergence* of equilibrium prices. In terms of complex dynamical systems, one might argue that both economists and policy makers frequently refer to a market as a *self-organising system* even though this aspect appears to be rather problematic from a modelling point of view. In the most basic setting a decentralised allocation of goods should therefore ideally support the interpretation of only two independent agents reaching an agreement over the price of a good.

The approach followed in this chapter is different from most of the existing literature on bargaining games. To the knowledge of the author, there exists no previous literature on the approach presented here. It starts from the correspondence between utility functions with a constant elasticity of substitution (CES) and the logit model in discrete choice literature as presented in Anderson et al. (1992). Unlike the discussion in Anderson et al. (1992) however, the immediate focus will not be on a representative consumer and differentiated goods. Standard textbook examples of a two-agent exchange economy often use a simple utility specification for illustration, such as a Cobb-Douglas function. Because the Cobb-Douglas function has a CES equal to one, this basic example of a general equilibrium model can also be translated to a discrete choice problem. A discrete choice formulation has three benefits in terms of a behavioural interpretation:

1. the agents’ choices can be interpreted as simple rule-based decisions,
2. the discrete set of choices can be assigned pay-offs that resemble those in game theory,
3. if the pay-offs depend only on the negotiated prices, bargaining can be identified as a strategic market interaction using a different interpretation of the original preference structures.

It is to be noted in advance that the game-theoretical interpretation above is distinct from what is usually called the *bargaining game* in economics (Nash, 1950; Rubinstein, 1982), although the type of negotiation suggested bears some similarities with Yildiz (2003). The Nash equilibria in the model developed in this chapter are rather a reinterpretation of the traditional market equilibrium in terms of *best responses* with a negotiated market price as a means for interaction. Two goals can be achieved by identifying the neoclassical market equilibrium with a Nash equilibrium, :

1. the theories on evolutionary approaches to game theory—especially those of *learning in games*—can be translated directly to the *computation* of an market equilibrium,
2. the interpretation of a Nash equilibrium can be sustained to allocations that are inefficient in the neoclassical framework.

The second goal will play an important role in the assessment of externalities as *non-market interactions*.

This chapter follows a stepwise approach to the development of an agent-based computational bargaining model. Section 4.2 discusses the efficiency of the market allocation following the traditional approach. In section 4.3 the CES utility function will be interpreted in terms of a best response to a given price. This best response is integrated in a dynamics which involves an institution setting disequilibrium prices in section 4.4. The institution is removed in section 4.5. In sections 4.6 and 4.7 external effects and product differentiation will be introduced briefly for this two-agent model. Conclusions are drawn in section 4.8.

4.2 Market efficiency

The First Theorem of Welfare Economics implies that, under certain conditions, if goods are allocated by competitive markets there exists a unique set of prices ensuring an optimal level of well-being for all agents. In a competitive economy it is assumed that all agents are price takers, meaning that no agent can individually influence the prices of goods sold in the markets. The optimality of the level of well-being is characterised as *Pareto efficient*, meaning that no agent ’... can be made better off without someone being made worse off...’ (Stiglitz, 2000, p. 57). Following Bowles (2004), this theorem can be interpreted with some scepticism. By means of a sort of *reverse engineering*, it can be shown that the condition of price taking behaviour might be interpreted as a mechanism that forces the agents to have identical marginal rates of substitution (MRS). In the neoclassical general equilibrium model the MRS is identical to the market price. And since it can be shown that agents having the same MRS already implies Pareto efficiency, it is the assumption that agents face the same price which effectively enforces the efficiency. This observation does not dismiss the first welfare theorem as a normative result. Rather, it shows that Pareto efficiency can be achieved independent from how market prices are established[1].

4.2.1 Equal marginal rates of substitution

The following assessment is adapted from Bowles (2004, p. 205–232). Given an economy with two agents and two goods, a Pareto efficient allocation can be defined as the market equilibrium in which agent 1 tries to solve the following problem:

$$\begin{aligned} &\max_{s_1, z_1} u_1\left(s_1, z_1\right) \\ s.t. \quad & u_2\left(s_2, z_2\right) = u_2^* \\ & s_1 + s_2 = \tilde{s} \\ & z_1 + z_2 = \tilde{z}. \end{aligned} \tag{4.2.1}$$

It is similar to an ordinary optimisation problem in which an agent tries to maximise utility given a budget constraint. Instead of a budget constraint, the agent now faces three constraints. The first constraint in (4.2.1), is the level of utility of agent 2 that needs to be remained fixed for Pareto efficiency[2]. The second and third constraint reflect the total availability—and thereby the total consumption—of the goods s and z.

Combining all three constraints, the utility function for agent 2 can be written as a function of the consumption of agent 1: $u_2\left(\tilde{s} - s_1, \tilde{z} - z_1\right) = u_2^*$. If it is furthermore assumed that this function can be inverted[3], the quantity z_1 can be written as a function of the variable s_1 and the constants u_2^*, $\tilde{s}$ and $\tilde{z}$: $z_1 = \tilde{z} - z_2\left(\tilde{s} - s_1; \tilde{z}, u_2^*\right) = z_1\left(s_1\right)$. Maximisation of utility in the problem of (4.2.1) can now be addressed with only a first-order condition (using the chain rule):

$$\frac{\partial u_1}{\partial s_1} + \frac{\partial u_1}{\partial z_1}\frac{dz_1}{ds_1} = 0. \tag{4.2.2}$$

Using the definition of the *marginal rate of substitution* (MRS) (Simon and Blume,

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[1] The fact that the neoclassical framework does not specify the process of price setting has inspired in the 20^{th} century economists like Lerner (1946) to explore the efficiency of centrally planned economies.

[2] The equality sign in the first constraint in (4.2.1) should actually be a $\leq$, but for simplicity it assumed here that the constraint is binding.

[3] The conditions for inversion correspond to the conditions for a convex preference ordering; the second postulate quoted as formulated by Koopmans (1957) in section 1.3.

1994, p. 348), it follows from (4.2.2) that

$$MRS_1 \equiv -\frac{\partial u_1/\partial s_1}{\partial u_1/\partial z_1} = \frac{dz_1}{ds_1}. \tag{4.2.3}$$

Solely based on the second and third constraint of (4.2.1), the following relations can be derived:

$$\frac{dz_1}{ds_1} = \frac{d}{ds_1}\left(z^* - z_2\right) = -\frac{dz_2}{ds_1}, \tag{4.2.4}$$

and finally

$$\frac{dz_2}{ds_2} = \frac{dz_2}{ds_1}\frac{ds_1}{ds_2} = \frac{dz_2}{ds_1}\frac{d}{ds_2}\left(\tilde{s} - s_1\right) = -\frac{dz_2}{ds_1} = \frac{dz_1}{ds_1}. \tag{4.2.5}$$

From (4.2.5) it follows that the marginal rates of substitutions are identical. The MRS reflects the marginal amount of one good an agent is willing to exchange for another, while keeping the same level of utility. This meaning is, however, of minor importance in this section. More important is that in the standard utility maximisation problem,

$$\begin{aligned} &\max_{s_1, z_1} u_1\left(s_1, z_1\right) \\ &\qquad\qquad s.t. \quad y_1 = p_s s_1 + p_z z_1, \end{aligned} \tag{4.2.6}$$

the budget constraint allows for the same type of substitutions as the constraints in (4.2.1), allowing the quantity z_1 to be written as a function of s_1. Therefore

$$MRS_1 = \frac{dz_1}{ds_1} = -\frac{d}{ds_1}\left(\frac{y_1 - p_s s_1}{p_z}\right) = -\frac{p_s}{p_z} = -p. \tag{4.2.7}$$

The MRS is identical to the negative price ratio for the individual agent. Price taking behaviour, as the fundamental characteristic of a competitive market, is captured in the budget constraint of (4.2.6), where it is assumed that the agent maximises utility at given prices. If all agents optimise their utility at the same given prices, all agents will have the same marginal rate of substitution. And all agents having the same marginal rate of substitution follows from the condition that the equilibrium must be Pareto efficient according to the solution to problem (4.2.1).

4.2.2 Linear spaces

The role of linear algebra in the efficiency of the allocation in the neoclassical framework discussed in chapter 3 will highlighted using the standard problem of (4.2.6).

In microeconomic theory, an agent faces a decision problem that in its simplest representation concerns the choice between two goods. Rather than just choosing the goods, the agent has to decide how to spend her income, or how to divide her income over the two goods. Following the tradition in urban economics, the standard problem will be cast in the example of a single agent who decides how to spend her income y over residential space or housing, s, and all other goods, z, the composite good or consumption bundle:

$$y = z + ps. \tag{4.2.8}$$

The price for space, the rent, amounts to p Euro per month. The price for all other goods is difficult to establish. Instead, z is measured in Euros itself, representing all the money spent on all other goods (see also Varian, 2003). The price ratio of (4.2.7) can therefore be read as the price for s. The choice of how much the consumer prefers to spend on each good is derived here in a less general approach than that usually found in economics textbooks. Instead of defining the consumption space in two dimensions, the budget constraint will be substituted from the start, as in section 4.2.1. As a result, the consumption space is one-dimensional[4]. If it is assumed that the agent does not save any money, the amount of all other goods she buys follows directly from the budget constraint:

$$z = y - ps. \tag{4.2.9}$$

Therefore, the choice reduces to the question of how much space, s, the agent prefers to rent every month.

In theory the agent could decide to spend all her money on either housing space, or to spend everything on all other goods. In practise, she will try to find a balance. Finding a balance might be thought of as finding a preferred point, s^*, on the line $\left[0, \frac{y}{p}\right]$. A criterion is needed that reflects the way the agent expresses what amount of housing space she prefers. A simple—and often criticised—way of representing the decision rule the agent adopts is introducing a function u of s over the interval $\left[0, \frac{y}{p}\right]$. The fact that agent i for example prefers the amount $s_{i,2}$ over the amount $s_{i,1}$,

$$s_{i,2} \succ s_{i,1}, \tag{4.2.10}$$

is then represented by

$$u\left(s_{i,2}\right) > u\left(s_{i,1}\right). \tag{4.2.11}$$

[4] Although this approach does not justice to the subtleties of general equilibrium analysis, it serves here only as an illustration of the role of linear algebra in the efficiency of the market equilibrium in neoclassical economics.

The optimal amount of space, from the perspective of the individual agent, can be expressed as the amount $s^*_{i,j}$ she prefers over all other amounts k. In mathematical terms:

$$u\left(s^*_{i,j}\right) > u\left(s_{i,k}\right) \qquad s^*_{i,j}, s_{i,k} \in [0, y/p] \qquad s^*_{i,j} \neq s_{i,k}. \tag{4.2.12}$$

For the optimal amount, $s^*_{i,j}$, to be unique, the original utility function, u, needs to be *convex* in s and z. With z already substituted for from the budget constraint, this condition is equivalent to the requirement that utility u is a *concave* function in s. One possible specification of u that fulfils the requirement is

$$u = sz. \tag{4.2.13}$$

With z substituted from the budget constraint, it follows that

$$u = s\left(y - ps\right) = -ps^2 + sy \tag{4.2.14}$$

This is simple hyperbolic equation that reaches its maximum exactly in the middle: $s^* = \frac{y}{2p}$. A more general specification is

$$u = s^\beta z^{1-\beta}. \tag{4.2.15}$$

In the literature, this expression is known as the *Cobb-Douglas* utility function. With the expression for z substituted in the last equation, a general specification for $u\left(s\right)$ is derived:

$$u\left(s\right) = s^\beta \left(y - ps\right)^{1-\beta}. \tag{4.2.16}$$

An example of this curve is plotted in figure 4.1. The more common approach is illustrated in figure 4.2. In figure 4.2 the utility curve touches the budget constraint. Since the slope of the budget constraint expressed as $z(s)$ in (4.2.9) is equal to $-p$, this figure illustrates the maximisation of utility and the equivalence of the MRS and the negative price as in (4.2.7). For two identical agents, the market equilibrium is presented in an *Edgeworth box* in figure 4.3. In figure 4.3 the basic intuition behind the First Welfare Theorem and the conjunction with the postulates formulated by Koopmans (1957), cited on page 16, is presented for an idealised symmetric case. Convexity guarantees strict separation of the consumption spaces of the two agents. The separation line in figure 4.3 is identical to the budget constraint for both agents. As discussed in section 4.2.1, at the point where the agents maximise their utility the slope—and by consequence the MRS—is the same. With a price on the budget curve for both agents that corresponds to this point, the agents are independently able to maximise their utility.

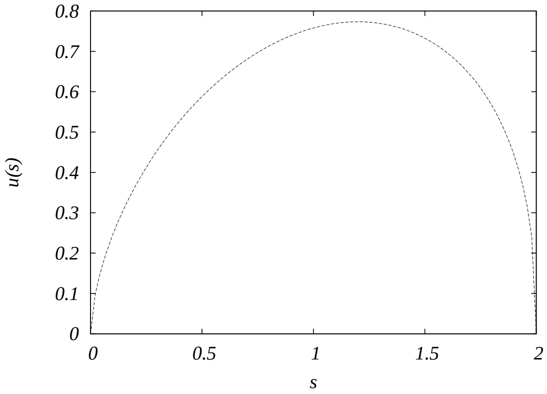

Figure 4.1: *Cobb-Douglas function* ($\beta = 0.6$, $y = 1.0$ *and* $p = 0.5$).

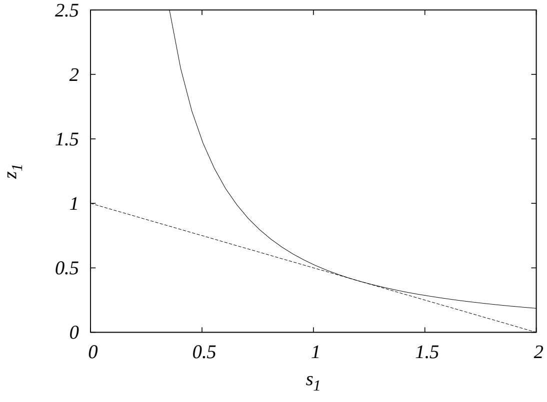

Figure 4.2: *Same Cobb-Douglas function as in figure 4.1, presented in the usual way.*

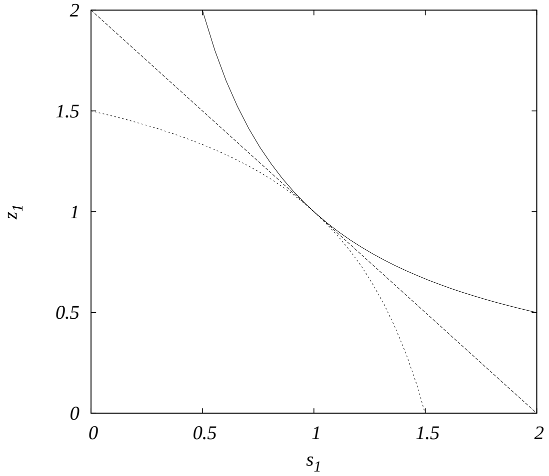

Figure 4.3: *Two Cobb-Douglas functions and budget curves ($\beta = 0.5$, $y = 2$ and $p = 1.0$).*

4.2.3 Benchmark exchange economy

An analytical solution for a simple specification for a two-agent, two-good model will be presented in the final part of this section, adapted from Varian (2003, p. 93–94). It serves to clarify the role of the Walrasian Auctioneer who sets the market clearing prices. Additionally, it will serve as a benchmark for the computational agent-based models developed in sections 4.4 and 4.5. The neoclassical general equilibrium model also includes production, but here an exchange economy is assumed. Consider an agent, i, facing the following problem:

$$\max_{s_i, z_i} u_i \quad s.t. \quad y_i = p_s s_i + p_z z_i. \tag{4.2.17}$$

It is assumed that the agent has a preference structure that conforms to a Cobb-Douglas utility function,

$$u_i = u\left(s_i, z_i; \beta_i\right) = s_i^{\beta_i} z_i^{1-\beta_i}. \tag{4.2.18}$$

Using the approach sketched in section 4.2.2, maximisation of the utility function can be performed by substituting the budget constraint first. The demand for land follows from a simple first order condition

$$\begin{aligned}\frac{du_i}{ds_i} &= \frac{d}{ds_i}\left[s_i^{\beta_i}\left(\frac{y_i - p_s s_i}{p_z}\right)^{1-\beta_i}\right] \qquad (4.2.19)\\ &= s_i^{\beta_i - 1}\left(\frac{y_i - p_s s_i}{p_z}\right)^{-\beta}\left[\beta_i\left(\frac{y_i - p_s s_i}{p_z}\right) - \frac{p_s}{p_z}\left(1-\beta_i\right)s_i\right] = 0.\end{aligned}$$

The demand for land is

$$s_i = \frac{\beta_i y_i}{p_s}, \qquad (4.2.20)$$

and the demand for all other goods equals

$$z_i = \frac{\left(1-\beta_i\right)y_i}{p_z}. \qquad (4.2.21)$$

The income of the agents results from only selling a part of their initial endowments in the two goods to each other at market prices.

$$y_i = p_s\omega_{s,i} + p_z\omega_{z,i}. \qquad (4.2.22)$$

The vector for the distribution of endowments can be written as

$$\underline{\omega}_i = \left(\omega_{s,i}, \omega_{z,i}\right), \qquad (4.2.23)$$

with $\omega_{s,1} + \omega_{s,2} = A$ and $\omega_{z,1} + \omega_{z,2} = 1$. The endowment vector for agent 1 will be specified as

$$\underline{\omega}_1 = \left\{w_s A, 1 - w_z\right\}. \qquad (4.2.24)$$

For agent 2 the vector is

$$\underline{\omega}_2 = \left\{\left(1-w_s\right)A, w_z\right\}. \qquad (4.2.25)$$

Here, the parameters w_s and w_z control the distribution between the two agents. Based on the demand function (4.2.20), total consumption at market prices will need to be equal to the sum of the initial endowments for both goods. This role of assumption from a mathematical perspective is clear though its behavioural interpretation is not, as will be discussed in subsequent sections.

For land, the following equation will need to be solved:

$$\begin{aligned}s_1 + s_2 &= \frac{\beta_1 y_1 + \beta_2 y_2}{p_s} \qquad (4.2.26)\\ &= \frac{\beta_1 w_s A p_s + \beta_1\left(1-w_z\right)p_z + \beta_2\left(1-w_s\right)A p_s + \beta_2 w_z p_z}{p_s} = A.\end{aligned}$$

The resulting equilibrium price is

$$p^* = \frac{p_s^*}{p_z^*} = \frac{\beta_1 (1 - w_z) + \beta_2 w_z}{A [1 - \beta_1 w_s A - \beta_2 (1 - w_s)]}. \tag{4.2.27}$$

With (4.2.27) as the solution for (4.2.26), the main behavioural interpretation lacking might be referred to as the question of who sets the equilibrium prices. By means of *ad hoc* explanation, an external institution could be introduced. This institution is often referred to as the metaphorical ‘Walrasian Auctioneer’ in economic theory, named after the French economist Léon Walras (1834-1910).

This example serves as an illustration of the main difficulty with the assumption of the neoclassical framework to be addressed in the following sections. Although utility maximisation might be regarded as a less than ideal assumption for modelling human behaviour, it can be regarded as a stylised formalisation of an agent having well-defined preferences. The main concern with market clearing—that is, solving for the equilibrium prices in 4.2.26—is that it has no behavioural connotation. In order to be able to address the issue of market clearing, first an alternative interpretation of utility maximisation will be introduced in the next section.

4.3 CES utility and best response

In this section it will be shown that the neoclassical economic behavioural assumption of utility maximisation can alternatively be interpreted as a strategic decision concerning the allocation of the budget directly. This interpretation is restricted to a specific class of utility functions, but this class is very general and covers many cases. It allows for the direct use of the indirect utility function, although in an alternative, stochastic interpretation. The main benefit of this interpretation is, however, not the possibility to use the indirect utility function without referring to the maximisation of the direct utility function first. *Rather, it allows for an identification of the strategic element in the budget allocation problem directly with a best response correspondence in game theory, discussed in chapter 3.* This theme will be developed further in chapter 6, because the best response correspondence is fundamental in the evolutionary approach to the concept of a Nash equilibrium that will be used to characterise the spatial equilibrium resulting from the location choices in a population consisting of many agents. First the relation between the traditional market equilibrium and the related type of Nash equilibrium in an exchange economy with only two agents will be explored in this chapter.

The reference problem will be the standard maximisation of a utility function as in 4.2. The utility function is assumed to have a *constant elasticity of substitution*

(CES) specification over two goods,

$$u\left(s,z\right)=\left[\beta s^{\rho}+\left(1-\beta\right)z^{\rho}\right]^{1/\rho}. \tag{4.3.1}$$

A CES function can be considered a generalisation of the Cobb-Douglas function, as will be shown in section 4.3.1. The budget constraint is given by (4.2.8). Using standard solution techniques, the demand for space can be shown to be

$$s=\frac{\beta^{\sigma}p^{-\sigma}}{\beta^{\sigma}p^{1-\sigma}+\left(1-\beta\right)^{\sigma}}y, \tag{4.3.2}$$

with $\sigma = 1/(1-\rho)$ as the *elasticity of substitution*. Following Anderson et al. (1992), the elasticity of substitution will be shown to be related to the variance of the error term of a random utility model (RUM) in the *logit* specification of a discrete choice model (McFadden, 1973, 1984). In this chapter this relation will not be employed in a model of product differentiation and oligopoly immediately, as in the examples presented by Anderson et al.. Instead, the relation between the interpretation of a RUM in a smoothed best response correspondence in learning in games (Fudenberg and Levine, 1998, p. 107-119) and the ordinary CES function for two goods (4.3.1) will be investigated. Although the discussion can be applied to any context with two goods, the example started in section 4.2 referring to *land* will be continued here.

4.3.1 Preference structure

The alternative problem formulation that will be shown to yield identical results as (4.2.6) with (4.3.1) deals with a stochastic decision problem for one agent. First, the budget constraint can be written as

$$y=x\left(ps\right)+\left(1-x\right)z. \tag{4.3.3}$$

The agent has to decide what fraction, x, of budget y to spend on residential space, or housing. The set of stochastic problems that corresponds to maximisation of (4.3.1) for all values of σ is based on the following non-stochastic decision:

$$\begin{aligned}&\text{if}\quad\left(\frac{a_s}{a_z}>p\right)\quad\text{then}\quad x=1,\\&\text{if}\quad\left(\frac{a_s}{a_z}\leqslant p\right)\quad\text{then}\quad x=0.\end{aligned} \tag{4.3.4}$$

In the condition,

$$\frac{a_s}{a_z}>p, \tag{4.3.5}$$

the ratio of a_s and a_z is the main factor in a basic preference structure. If the initial preference for space, a_s, relative to the initial preference for all the other goods,a_z, exceeds the price for housing, p, the agent chooses to spend his entire budget on housing. Using (4.3.4) has the advantage that the behavioural model of the agent is reduced to a *rule-based* decision. Whereas a rule-based system is often used for implementing an agent-based model, here it serves first as the minimal basis of an elementary theoretical model of human behaviour. In this basic form—without a stochastic component—, the decision rule can generate only two demand functions. If $x = 1$, it would follow that $s = y/p$, while if $x = 0$, the entire budget would be spent on all other goods than housing and therefore $z = y$ and $s = 0$. In terms of the discussion on utility maximisation in section 4.2.2, only the extremes of the interval $\left[0, \frac{y}{p}\right]$ can be chosen by the agent.

To introduce a more general, stochastic problem a parameter ν will be added[5]. The general condition, of which (4.3.5) is a special case, can be stated as

$$\nu^{\mu} a_s s > (1-\nu)^{\mu} a_z z, \tag{4.3.6}$$

with $\nu \in [1, 0]$ from a *uniform* distribution and μ as a parameter controlling the 'amplitude' of the stochastic weights for the preference factors a_s and a_z. For s and z in (4.3.6) the extreme cases, $s = y/p$ and $z = y$ can be substituted. Because the income term, y, would appear on both sides of (4.3.6), it does not play a role in the decision. With $a \equiv a_s/a_z$ the condition (4.3.6) reduces to

$$e^{\mu\varepsilon} a > p, \tag{4.3.7}$$

with

$$e^{\varepsilon} \equiv \left(\frac{\nu}{1-\nu}\right). \tag{4.3.8}$$

The term (4.3.8) marks the only difference between (4.3.5) and (4.3.6). It also captures the stochastic weighting factors in one new factor, ν, with a well-known distribution. From the definition (4.3.8) follows the cumulative probability distribution for ε. Based on the draws from the uniform distribution for ν, the cumulative distribution for ε—as the probability that ν has a certain value —can be written:

$$\nu = F(\varepsilon) = \frac{e^{\varepsilon}}{e^{\varepsilon}+1}. \tag{4.3.9}$$

---

[5] The introduction of ν is more of less arbitrary at this stage, except for the possibility of controlling the impact of the weights with a *single* parameter. The justification will follow in section 4.3.4.

This is known as the *logistic distribution*. A more general justification of the use of the logistic distribution will be given in section 4.3.4, since the ad hoc introduction of ν in (4.3.6) might seem arbitrary at this point. For now, it mainly facilitates a focus on the relation between the elasticity of substitution and the variance of the logistic distribution. In figure 4.4, the function $F(\varepsilon)$ from (4.3.9) is plotted, together with the logistic transformation of several random draws for ν. The fraction x will now be defined as the *probability* that the agent will spend his entire budget on housing. Taking the logarithms of (4.3.7) results in

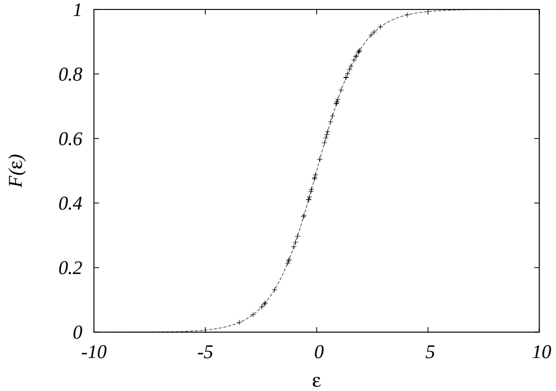

Figure 4.4: *Logistic distribution and logistic transformation of draws from a uniform distribution.*

$$\ln a - \ln p + \mu\varepsilon > 0. \tag{4.3.10}$$

Next, an evaluation function is defined[6] that will have a similar role as the implication $x = 1$ in decision rule (4.3.4). The evaluation function $I(\bullet)$ has a value of 1 if the condition within the brackets is *true* and a value of 0 if it is *false*. It can therefore be considered the formalisation of the decision rule (4.3.4). Evaluation of (4.3.10), together with the probability density $f(\varepsilon)$, based on (4.3.9), results in the *probability* that the condition is ‘true’ ($x = 1$ in (4.3.4)) as the *expected value*, or the mean, of $I(\bullet)$:

$$x = \int_{-\infty}^{\infty} I\left(\varepsilon > -\frac{\ln a - \ln p}{\mu}\right) f(\varepsilon)\, d\varepsilon. \tag{4.3.11}$$

[6] The introduction of the evaluation function and the derivation of (4.3.12) are based on Train (2003).

Performing the integration in (4.3.11) results in

$$x = \int_{-\frac{\ln a - \ln p}{\mu}}^{\infty} f(\varepsilon)\, d\varepsilon = 1 - \frac{\exp\left[-\left(\ln a - \ln p\right)/\mu\right]}{\exp\left[-\left(\ln a - \ln p\right)/\mu\right] + 1}$$

$$= \frac{\exp\left[\left(\ln a - \ln p\right)/\mu\right]}{\exp\left[\left(\ln a - \ln p\right)/\mu\right] + 1}$$

$$= \frac{(a/p)^{1/\mu}}{(a/p)^{1/\mu} + 1}$$

$$= \frac{(a_s/p)^{1/\mu}}{(a_s/p)^{1/\mu} + a_z^{1/\mu}}. \qquad (4.3.12)$$

The probability is now expressed as a function of the preference factors, a_s and a_z, and the price of housing, p.

Only a redefinition of the constants is needed: $1/\mu \equiv \sigma - 1$, $\beta \equiv a_s^{(\sigma-1)/\sigma}$ and $(1-\beta) \equiv a_z^{(\sigma-1)/\sigma}$. Using these new constants, the probability can be rewritten as

$$x = \frac{\beta^\sigma p^{1-\sigma}}{\beta^\sigma p^{1-\sigma} + (1-\beta)^\sigma}. \qquad (4.3.13)$$

Finally, using (4.3.3), the demand for space is given by $s = x\frac{y}{p}$, or

$$s = \frac{\beta^\sigma p^{1-\sigma}}{\beta^\sigma p^{1-\sigma} + (1-\beta)^\sigma} \frac{y}{p}, \qquad (4.3.14)$$

which is identical to (4.3.2).

As in Anderson et al. (1992), the elasticity of substitution σ is shown to be related to the ‘amplitude’ or variance μ of the stochastic term ε. Here, it is also shown that the more or less heuristically defined preference factors, a_s and a_z, in the condition (4.3.6) are related to the parameter β in the equivalent CES utility function. It can therefore be concluded that a rule-based decision with these factors and the price p in the condition (4.3.10) results in a demand for housing, (4.3.14), through the ‘strategy’, x, expressed in (4.3.13), without solving a traditional constrained optimisation problem. In figures 4.5 and 4.6 the probability and the demand functions respectively, are plotted with $\sigma = 0.3$ and $\beta/(1-\beta) = 0.5$.

4.3.2 Probability as time average

Using the stochastic interpretation of the decision made by the agent means that the notion of a *fraction* of the income spent on housing is replaced by the *proba-*

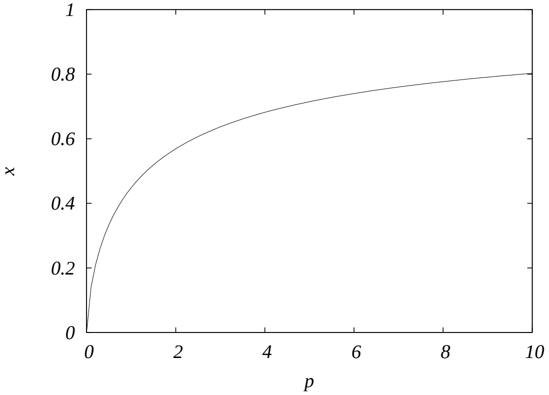

Figure 4.5: *Probability cf.* (4.3.13).

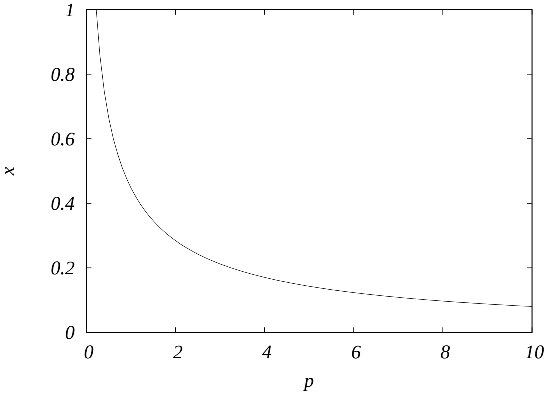

Figure 4.6: *Demand cf.* (4.3.14).

bility that the entire income will be spent on housing. For use in a regular economic decision problem, the original interpretation—referring to a fraction—is usually preferred. The apparent discrepancy between the two interpretations of x vanishes if the probability is interpreted as the theoretical ideal value of the *mean* based on a finite number of draws for ε. In this interpretation the income is divided into T parts. If, for example, it is assumed that y is the agent's monthly income, T would refer to the number of spendings in a month. For each part, the agent decides whether or not to spend it on housing. The decision is based on the ratio of the initial preference factors, weighted by a stochastic term, $e^{\mu\varepsilon}$, as in (4.3.7). The value of this term can be interpreted as unknown to the modeller, but known to the agent (see Fudenberg and Levine, 1998, p. 106). If the values of the each weight factor are known to the agent, the stochastic element would be transferred to the 'circumstances' at time t that can make housing more or less important than all other goods. The circumstances refer to all factors other than the price that might have an influence on the decision of spending an amount of money equal to y/T on housing.

Splitting the income into T parts and adding up all the parts spent on housing is equivalent to calculating the average amount of money spent on housing times T. In this way, the value of

$$\hat{x} = \frac{1}{T}\sum_{t=1}^{T} I\left(\frac{\ln a - \ln p}{\mu} > -\varepsilon_t\right), \tag{4.3.15}$$

would serve as an approximation to (4.3.11). Combining all considerations above, the part of the income that is spent—or will be spent—on housing can be represented by

$$p\hat{s} = \hat{x}y = \frac{y}{T}\sum_{t=1}^{T} I\left[\left(\frac{\nu_t}{1-\nu_t}\right)^{\frac{\rho}{1-\rho}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\rho}} > p\right]. \tag{4.3.16}$$

The demand $\hat{s}$ will be a close approximation of the demand in (4.3.14) if T is large. Both demand functions will be identical if $T \to \infty$.

Although this interpretation is relatively abstract, it is important to remember that the behaviour of the agent was essentially reduced to a single decision. If the fraction of the income spent on housing is reinterpreted as the average amount of money an agent—or household—spends on housing every month, it is clear that this decision will result from some process, strict bookkeeping or experience.

4.3.3 Special cases of CES

Since expenses for housing are separated from expenses on all other goods, the amplitude or variance of the stochastic term can also be interpreted as the stability with which the agent can sustain his preference for housing with respect to its market price. This interpretation relates the original concept of elasticity of substitution, σ, in a natural way to the notion of randomness, expressed in μ. By definition the following relations hold

$$\mu = \frac{1-\rho}{\rho} = \frac{1}{\sigma - 1}. \tag{4.3.17}$$

This allows for an interpretation of the variance, μ, of the stochastic term ε in at least two of the three special cases of the CES utility function (compare with Varian, 1992, p. 19). All three cases are summarised in table 4.1. The role of μ is in the first case less intuitive than in the other two. From (4.3.17) it follows that $\sigma = 0$ if $\mu = -1$. An elasticity of substitution equal to zero implies that the goods cannot be substituted. The relative preference for one good over the other, expressed in factor β, plays no role and both goods are always bought together. Specifically, the demand for housing will be equal to $s = y/(1-p)$, while the demand for the composite good is given by $z = 1/(1-p)$. Both demands therefore depend on the price for housing in the same way. This is consistent with the intuitive idea of perfect complementarity, but the interpretation of $\mu = -1$ does not add much to this insight.

1	$\sigma \downarrow 0$	$\rho \to -\infty$	$\mu \to -1$	*perfect complements*
2	$\sigma \to 1$	$\rho \to 0$	$\mu \to \infty$	*Cobb-Douglas*
3	$\sigma \to \infty$	$\rho \to 1$	$\mu \to 0$	*perfect substitutes*

Table 4.1: *Special cases of CES functions.*

In the second and third case, Cobb-Douglas and perfect substitutability, the variance μ does have a clear interpretation. These cases represent two extremes values of the variance. Starting with the third case, if $\mu = 0$ the stochastic term disappears in the condition (4.3.10). As a result, the two goods, housing and the consumption bundle, are only evaluated on the basis of preference factors and the price. The circumstances identified with the stochastic term have no impact on the decision. It suggests that both goods play an identical role in the consumption pattern of the agent and that they therefore can be treated as perfect substitutes when the preference factors, a_s and a_z in (4.3.5), are identical.

The second case results in a fraction of the income on which the price has no impact, as can be concluded from (4.3.13) if $\sigma = 1$. Total expenses on land will always be equal to a fraction β of the income. In terms of the variance of the stochastic terms, especially in (4.3.10), $\mu \to \infty$ reflects a situation of total randomness, where the circumstances completely overrule the impact of the price. This characteristic stresses the position of the Cobb-Douglas utility function as a highly stylised case.

The more general cases where the value of μ lies somewhere between 0 and ∞ represent situations in which circumstances play a limited or larger role. In addition to the alternative interpretation of utility maximisation as a sequence of rule-based decisions, the elasticity of substitutions can therefore be reinterpreted as an indicator for the impact of all factors other than the price in these decisions.

4.3.4 Indirect utility

For the addition of the stochastic term ν to the condition (4.3.6) of the decision rule no clear interpretation has yet been given. As was noted in section 4.3.3, the main focus there was on the role of the variance, μ, rather than on the origin of the error term, ε. The error term with a logistic distribution can be interpreted as the difference between two other error terms. Using the definition $\varepsilon \equiv \varepsilon_s - \varepsilon_z$, condition 4.3.6 can be rewritten as

$$a_s s e^{\mu\varepsilon_s} > a_z z e^{\mu\varepsilon_z}. \tag{4.3.18}$$

Instead of expression (4.3.10), the main condition in the decision rule—after taking logarithms on both sides of (4.3.18)—can be written as

$$\ln v_s + \mu\varepsilon_s > \ln v_z + \mu\varepsilon_z. \tag{4.3.19}$$

If the random terms ε_s and ε_z have a *double exponential distribution,* it is ensured that the difference between the two still has a logistic distribution (see Appendix). With $a_s \equiv \beta^{1/\rho}$ and $a_z \equiv (1-\beta)^{1/\rho}$ as before, the following definitions result for (4.3.19):

$$\ln v_s \equiv \ln y + \frac{1}{\rho}\ln\beta - \ln p, \tag{4.3.20a}$$

$$\ln v_z \equiv \ln y + \frac{1}{\rho}\ln(1-\beta). \tag{4.3.20b}$$

The choice based on the condition (4.3.19) can now be reinterpreted as finding the good, s or z, that yields the maximum value of the two elements of (4.3.20). This might be expressed as

$$q = \arg\max_{s,z}\left\{\ln v_s^{1/\mu} + \varepsilon_s, \ln v_z^{1/\mu} + \varepsilon_z\right\}, \tag{4.3.21}$$

followed by $x = 1$ if $q = ps$ and $q = z$ if $x = 0$. This notation is rather cumbersome and does not give any new information. However, (4.3.20) does contain additional information. The *expected value of the maximum* is (see Appendix)

$$\begin{aligned}\ln v &\equiv \mathcal{E}\left[\max\left\{\ln v_s^{1/\mu} + \varepsilon_s, \ln v_z^{1/\mu} + \varepsilon_z\right\}\right] \\ &= \mu \ln\left[\exp\left(\ln v_s^{1/\mu}\right) + \exp\left(\ln v_s^{1/\mu}\right)\right]. \qquad (4.3.22)\end{aligned}$$

According to this definition, the value of v corresponds exactly to the *indirect utility function* that belongs to the original maximisation problem of (4.2.6) with a CES utility specification according to (4.3.1):

$$v = y\left[\beta^\sigma p^{1-\sigma} + (1-\beta)^\sigma\right]^{1/(\sigma-1)}. \qquad (4.3.23)$$

This section proves that the expected level of utility for the interpretation derived in this chapter is identical to the indirect utility function that belongs to (4.3.1).

Due to the direct relation with the alternative formulation for the elements in the condition (4.3.18), the use of the indirect utility function is consistent with a rule-based interpretation of the individual agent's decision. In this sense, the traditional indirect utility can be used without an appeal to the maximisation of the direct utility as a behavioural assumption. This indirect utility function will be used for welfare analyses in later chapters.

4.3.5 Best response and CES

Although the decision of what fraction of the income to spend on housing was already referred to as a 'strategy', a real strategy demands interdependent decision-making, in short *interaction*. Conforming to the neoclassical tradition, a strict separation can now be made between market and non-market interactions. Market interactions are mediated through prices. Before addressing who will determine the prices, the following assumption will be made concerning the price for housing:

$$pA = x_1 y_1 + x_2 y_2. \qquad (4.3.24)$$

This expresses the value of land as the product of price per surface area and the total surface area available on the left-hand side. On the right-hand side are the fraction of the income spent on land times the income for agent 1 and 2 respectively. To focus the discussion on the strategic interaction between the agents, the existence of an institution similar to the Walrasian Auctioneer might be assumed at this stage. The main difference with the traditional auctioneer is the validity of (4.3.24) as a *disequilibrium price*. Given the total amount of money offered for land, the price is calculated as the value per surface area unit. The task of the auctioneer reduces to collecting all bids and dividing the sum of the bids by the total amount of land available. This price will then be quoted by the auctioneer. A more detailed discussion on this *myopic auctioneer* appears in section 4.4.

For finding a best response to any quoted price, the current strategy of agent 1 will be rewritten as

$$x_1 = \frac{\beta_1^{\sigma_1} p^{1-\sigma_1}}{\beta_1^{\sigma_1} p^{1-\sigma_1} + (1-\beta_1)^{\sigma_1}} = \frac{\beta_1^{\sigma_1}}{\beta_1^{\sigma_1} + (1-\beta_1)^{\sigma_1} p^{\sigma_1 - 1}}. \tag{4.3.25}$$

This expression can be used for solving for the price to which x_1 would be the best response:

$$p = \left(\frac{\beta_1}{1-\beta_1}\right)^{\frac{\sigma_1}{\sigma_1-1}} \left(\frac{1}{x_1} - 1\right)^{\frac{1}{\sigma_1-1}}. \tag{4.3.26}$$

Combining (4.3.24) and (4.3.26) results in

$$x_2 = \frac{1}{y_2}\left[A\left(\frac{\beta_1}{1-\beta_1}\right)^{\frac{\sigma_1}{\sigma_1-1}} \left(\frac{1}{x_1} - 1\right)^{\frac{1}{\sigma_1-1}} - x_1 y_1\right]. \tag{4.3.27}$$

The interpretation of (4.3.27) requires additional explanation. It is the strategy played by agent 2 to which the best response would be strategy x_1, played by agent 1. Stated differently, (4.3.27) is the 'implicit' best response x_1 to strategy x_2 and should actually be inverted. Unfortunately, due to its functional form, an analytical expression for x_1 as a function of x_2 cannot be obtained and (4.3.27) will need to be solved numerically[7]. Nevertheless, the function can be plotted followed by an inversion of the axes to gain at least a qualitative interpretation. This approach is followed in figure 4.7. The plot shows that the best response x_1 is increasing monotonically in x_2 the give parameters. If it is assumed that both agents are identical, the Nash equilibrium can be obtained at least graphically by finding the intersection of both best response curves. This is done in the second plot (figure 4.8). The second plot can be interpreted in exactly the same way as the best response plots in game theory, discussed in chapter 3. Section 4.5 will show that this Nash equilibrium corresponds to market clearing demands. Therefore, by using the interpretation of a CES utility function as introduced in this chapter, a Nash equilibrium in fractions of income spent on a good is defined that corresponds to the neoclassical market equilibrium.

4.4 The myopic auctioneer

The price setting institution that quotes the disequilibrium prices based on current bids was labelled 'myopic auctioneer' in the previous section. Unlike the auction-

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[7] The solution might be found using the *bisection method* (Press et al., 2002).

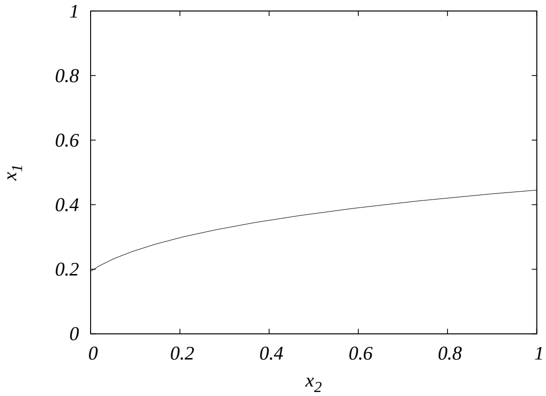

Figure 4.7: *Best response.*

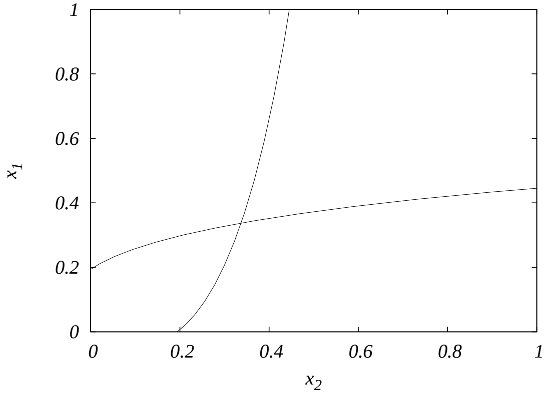

Figure 4.8: *Nash equilibrium (best response to a best response).*

eer in the traditional neoclassical framework, the myopic auctioneer need not have any knowledge about the preference structures of the agents involved in order to calculate the equilibrium prices that clear the markets. Instead, he can *mechanically* calculate the price at the current time step, based on assumed revealed bids from the two agents. Although this still might be considered a rather stylised description of real-world price formation, it does not require extreme cognitive capabilities of all parties involved. The process can be thought of as metaphor for trial and error. In this sense, it abstracts from a possible dynamics that eventually leads to a *stationary* equilibrium. It can be considered as an example of *evolutionary dynamics*, although a similar concept is known in neoclassical economics as *tatônnement process*. The traditional tatônnement process is based on raising prices in case of excess demand and lowering prices in case of excess supply. Since total supply is fixed in an exchange economy, the simple discrete dynamics for the myopic auctioneer based on (4.3.24) can be written as

$$p^t = \frac{x_1^{t-1} y_1 + x_2^{t-1} y_2}{A} = p^{t-1} \frac{s_1^{t-1} + s_2^{t-1}}{A}. \tag{4.4.1}$$

The equilibrium price is reached if $p^t = p^{t-1} = p^*$. In that case, from (4.4.1) it follows that

$$p^* \left(A - s_1^* - s_2^*\right) = 0, \tag{4.4.2}$$

which is known as *Walras' Law*, formally stating that the value of excess demand is equal to zero (Varian, 1992, p. 317). If $p^* > 0$, (4.4.2) expresses that the market is cleared, $s_1^* + s_2^* = A$, or that there is an equilibrium between supply and demand.

4.4.1 Chaotic dynamics

Once dynamics are introduced, the question arises of whether the process converges to the equilibrium described above. Computational experiments with two agents with a demand function (4.3.14) introduced in section 4.3 together with a myopic auctioneer show the possibility of the existence of a chaotic time series for certain parameter values, much in spirit with the logistic equation discussed in chapter 3. This result bears similarities with related models used for finance (Brock and Hommes, 1997).

The nature of the chaotic time series can be illustrated in more detail by plotting the *attractor* (Strogatz, 1996) next to the time series. This is done in figures 4.9-4.13. Here, the demands from both agents are plotted for the iterations $1001 - 5000$. The first 1000 are skipped to make sure that the dynamics reaches a stable pattern in which combinations of demands cycle. The elasticity of substitution, σ, serves as

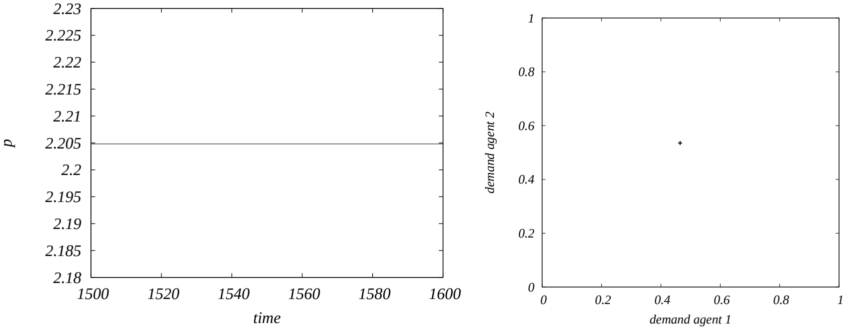

Figure 4.9: *Point attractor for* $w_1 = w_2 = 0.36$, $\sigma_1 = \sigma_2 = 1.0$.

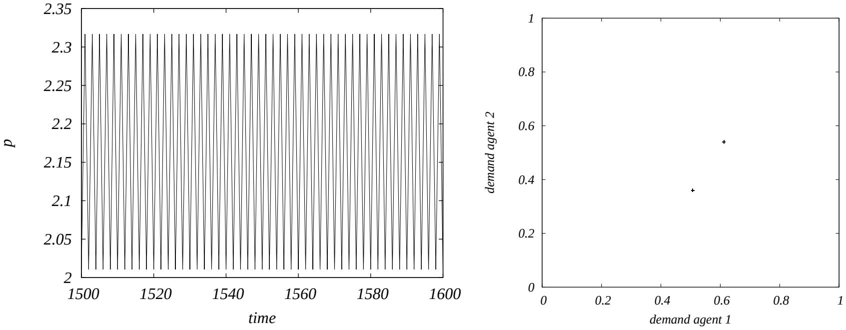

Figure 4.10: *Period doubling for* $w_1 = w_2 = 0.36$, $\sigma_1 = \sigma_2 = 7.0$.

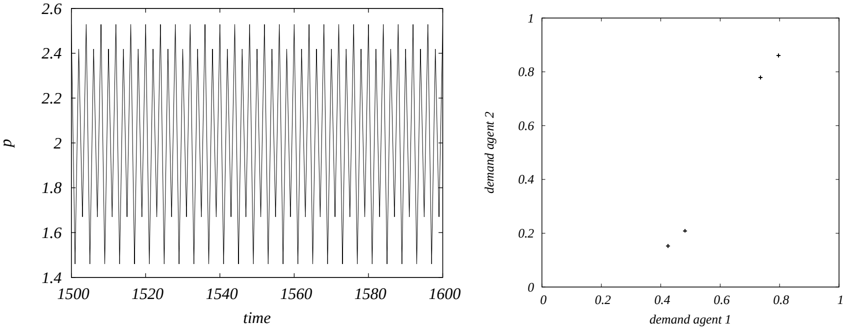

Figure 4.11: *Period four for* $w_1 = w_2 = 0.36$, $\sigma_1 = \sigma_2 = 10.0$.

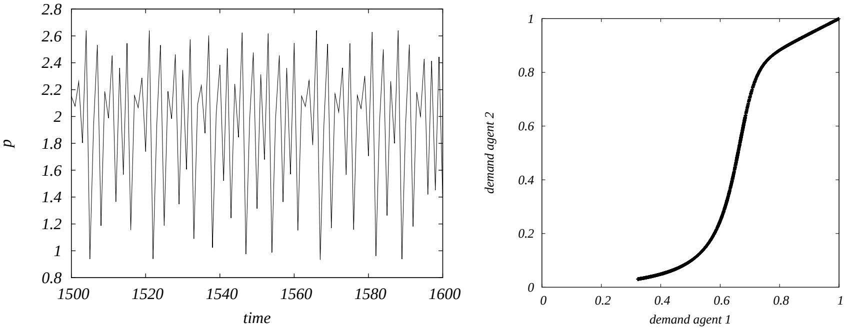

Figure 4.12: *Chaos and strange attractor for* $w_1 = w_2 = 0.36$, $\sigma_1 = \sigma_2 = 15.0$.

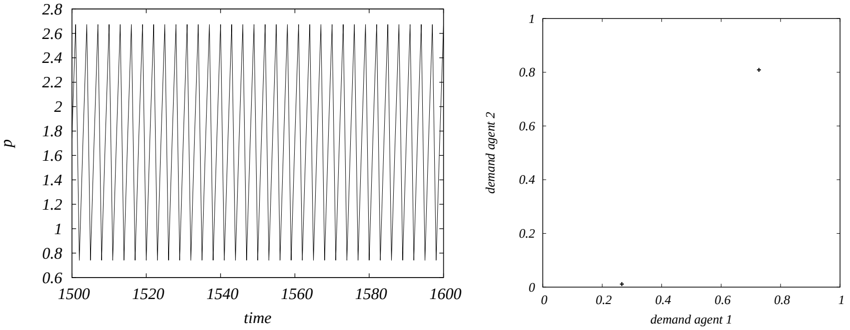

Figure 4.13: *Period two again for* $w_1 = w_2 = 0.36$, $\sigma_1 = \sigma_2 = 18.0$.

a *bifurcation parameter*. The number of combinations increases as σ increases and a dotted line with increasing density emerges (figure 4.12). This line is a so-called *strange attractor*. This strange attractor disappears again as σ is increased further (figure 4.13).

With the elasticity of substitution as the bifurcation parameter, the model suggests that the extent of volatility would increase as soon as the agents start to consider the two goods more and more as substitutes. Since a high value for σ implies a value for μ—the variance of the error term in the condition of the rule-based decision—that approaches zero, this result corresponds to use of an error term to smooth the best response (see chapter 3). A reducing impact of ‘circumstances’ other than the price, make the agents’ decisions very sensitive to price fluctuations. As a result of this sensitivity, the fluctuations are sustained. Another part of the explanation for the problem with convergence lies in the simultaneous reaction of both agents. This excludes the possibility of some sort of bargaining.

4.5 Bargaining variant

In this section, two individual behavioural models will be combined in a relatively simple bargaining model[8]. The main result is the possibility to show that the two agents of section 4.4 can agree in principle upon a price without the mediation of an auctioneer. Although the assumptions that need to be imposed on this bargaining

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[8] The model presented here was implemented by the author in the programming language *Python* (version 2.5). Python combined the advantages of a scripting language (no compilation) with object-orientation. It is therefore very convenient for building relative simple prototypes with a limited number of agents.

game may not be a very realistic description of a real bargaining process, the assumptions regarding the agents' level of rationality and access to information are not very demanding. The bargaining model is inspired by the smoothed fictitious play model presented in Fudenberg and Levine (1998, p. 28–36), in which agents form a belief concerning the other agent's preferred strategy. Fictitious play is both an example of an elementary *reinforcement learning* model in the literature on agent-based modelling as well as a bridge between evolutionary game theoretical concepts at the individual and the population level.

Imagine two agents bargaining over a piece of land of size A. A fraction w_s of the land is owned by agent 1. Agent 1 furthermore owns a fraction of $1 - w_s$ of the total sum of money. To make the example more concrete, it might be assumed that the total sum is equal to a certain amount in some currency, but for convenience the sum will be normalised to 1. The second agent, agent 2, owns a fraction $1 - w_s$ of the land and w_z of the money. Both agents would like to trade some part of their land in return for money, or money for land. They expect to benefit from the trade if they can agree upon a price per square meter and the amount of land they can trade. This example is identical to the benchmark problem of section 4.2.3, where the agents have a Cobb-Douglas utility function.

It will be assumed that each agent has a well-specified demand curve $s_i(p)$. This means that agents can express the amount of land they consider optimal as a function of the price per square meter. The way the agents determine the optimality of the amount of land and how they express their demand is based on the best response function, (4.3.25), that corresponds to a CES utility function in section 4.3.5. If an agent proposes a price per square meter of land, he implicitly also proposes the amount of land he prefers to buy by virtue of his demand curve. Therefore, every offer can be expressed as a price-quantity pair. The product of the two is exactly equal to the fraction of income the agent proposes to spend on land times his income.

The proposed part of his income one agent intends to spend on land will be observed by the other agent. Given the pair of price and amount proposed by the first agent, the second agent can determine the amount of land that would be left for him. He is expected to buy this amount at the price proposed by the first agent. However, if the combination of the amount left and the price proposed by the first agent does not match the demand curve of the second agent, the latter will decline the offer. If the second agent declines the offer, he will propose a new price. When determining a new offer, the agent considers the total amount of money the first agent intended to spend on land as given and fixed. Given this amount, the second agent calculates the price for the remaining amount pair that would match his own demand curve and will 'clear the market'. If agent 2 takes the amount of money agent 1 intended to spend on land as given, this amount can be expressed as h_1, where by definition

$$h_1 = p_1 s_1 = x_1 y_1. \tag{4.5.1}$$

Here, the index of the price indicates which agent made the price offer. Agent 2 will determine the pair of the price, p_2, and the amount of land, s_2, that will match both his demand curve, $s_2\left(p_2\right)$, and the condition

$$p_2 A - x_2 y_2 = h_1. \tag{4.5.2}$$

Next, agent 2 will propose the new price together with his preferred amount of land to agent 1. Agent 1 will only accept the offer if the pair consisting of p_2 and $s_1 = A - s_2$ matches his demand curve (4.3.14). If agent 1 declines, he can propose again, now taking the total amount of money agent 2 intended to spend on land as given using (4.3.27). This process with alternate offers will be repeated until both agents propose the same price and an agreement is reached.

Using the smoothed best-response function (4.3.27) derived in section 4.3.1 for the fraction of income to be spent on land, the bargaining problem sketched above can be interpreted as a best response dynamic, similar to reinforcement learning where agents learn to play a Nash equilibrium in a repeated game. In accordance with the previous analyses in this chapter, this Nash equilibrium corresponds to the neoclassical market equilibrium, which reflects a Pareto-efficient allocation. For the implementation of the procedure outlined above as an *agent-based model*, it should be noted that the only information that needs to be communicated between the agents consists only of the total amount of money the agent intends to spend on land,

$$h_i = p_i s_i = x_i y_i. \tag{4.5.3}$$

While making explicit the components p_i and s_i facilitates the interpretation as two agents bargaining over a price, the components of the alternative interpretation of h_i—the fraction x_i and the income y_i—instead primarily serves a theoretical purpose. It stresses the game theoretic quality of the interaction in determining the best responses x_i. Furthermore, in this second interpretation it can be shown that if an agreement is reached the solution is equivalent to a Nash equilibrium in *strategies* x_i. Nevertheless, the agents do not need to reveal the level of income, y_i; revealing p_i and s_i is sufficient. The dynamics of the price for this bargaining model are plotted in figures 4.14-4.16 for several parameter values. The oscillating price dynamics give an indication of the presence of a negative feedback mechanism with damping, stressing the possibility of adopting a systems perspective on this agent-based computational economic model. This is consistent with the methodological considerations in chapter 2. The results of simulation runs with this bargaining model can be compared with both the analytical result from the traditional solution strategy in section 4.2.3, which serves as a benchmark, and the simulation result from the myopic auctioneer model in section 4.4.

For comparison with the analytical solution, in figure 4.16 $\sigma_1 = \sigma_2 = 1.0016$ is chosen as an approximation as close as possible to $\sigma_1 = \sigma_2 = 1.0$ (Cobb-Douglas)

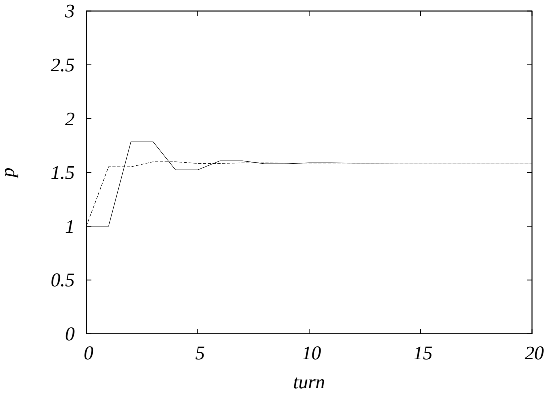

Figure 4.14: $\beta_1 = 0.75$, $\beta_2 = 0.6$, $\sigma_1 = 10.0$, $\sigma_2 = 20.0$, $w_1 = 0.3$, $w_2 = 0.4$.

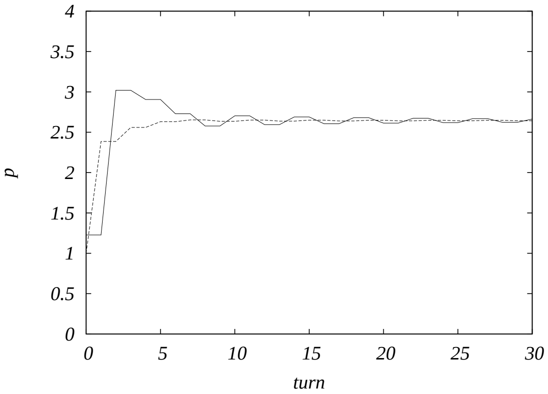

Figure 4.15: $\beta_1 = 0.75$, $\beta_2 = 0.6$, $\sigma_1 = \sigma_2 = 30.0$, $w_1 = 0.75$, $w_2 = 0.8$.

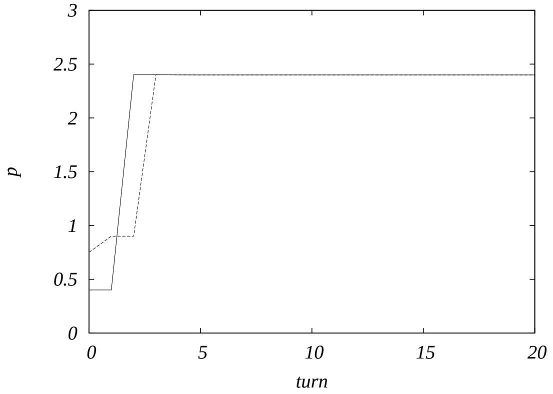

Figure 4.16: $\beta_1 = 0.75$, $\beta_2 = 0.6$, $\sigma_1 = \sigma_2 = 1.0016$, $w_1 = w_2 = 1.0$.

in order to avoid numerical problems (division by zero in the exponents of (4.3.27)) in figure 4.16. From (4.2.7), with $w_s = w_z = 1.0$, $\beta_1 = 0.75$ and $\beta_2 = 0.6$, the exact solution is $p^* = 2.4$, while the numerical approximation from the simulation gives $\hat{p}^* = 2.39992074$.

4.6 Product differentiation and network externalities

This section discusses the impact of network externalities on the Pareto efficiency in the traditional interpretation of the market equilibrium. Network externalities can be considered an example of non-market interactions. It also includes a short introduction on product differentiation and quality, because the presence of network externalities is particularly interesting for a differentiated product. The issues introduced in this section will return in chapters 6 and 7, where they will be discussed in more detail. The issues presented here are included in this chapter to highlight their relation with the basic notion of Pareto efficiency in the two-agent exchange economy and the game theoretical assessment of interaction discussed in chapter 3. It will be shown that a model with network externalities in principle can have multiple equilibria similar to the model of the coordination game presented in section 3.4.

4.6.1 Network externalities and Pareto efficiency

For simplicity, the discussion will be limited to the case of two identical agents with a Cobb-Douglas utility function. The utility function for agent i with a network externality, expressed by x_j, can be written as

$$u_i = s_i^{\beta} z_i^{1-\beta} x_j^{\gamma}. \tag{4.6.1}$$

The budget constraint is the same as before (given by (4.2.8)),

$$y_i = ps_i + z_i, \tag{4.6.2}$$

and will be used again for defining the demand s_j implicitly by the fraction of the income,

$$x_j = \frac{ps_j}{y_j}, \tag{4.6.3}$$

spent on s_j at a given market price p. The externality in (4.6.1) can be thought of as the fraction of the income the *other* agent spends on s. The choice of x over s is made for convenience, as will be shown below, but one can always be translated into the other using (4.6.3). Substitution of (4.6.3) in (4.6.1) shows that the consumption s_j of agent j has an impact on the utility level of agent i.

Only equilibrium solutions will be calculated in this section, without explicit reference to the evolutionary approach of section 4.5, though the results will apply for that approach as well. Furthermore, since the agents are assumed to be identical, in equilibrium $x_i = x_j = x$.

Elimination of z_i in (4.6.1) by using the budget constraint 4.6.2, followed by the maximisation of utility—with the externality taken as given—results in the following first-order condition:

$$\begin{aligned} \frac{du}{ds} &= \beta s^{\beta-1}\left(y-ps\right)^{1-\beta} x^{\gamma} - p\left(1-\beta\right) s^{\beta}\left(y-ps\right)^{-\beta} x^{\gamma} \\ &= s^{\beta-1}\left(y-ps\right)^{-\beta} x^{\gamma}\left[\beta\left(y-ps\right)-\left(1-\beta\right)ps\right] = 0. \end{aligned} \tag{4.6.4}$$

Solving results in

$$\beta\left(y-ps\right)-\left(1-\beta\right)ps = \beta y - ps = 0, \tag{4.6.5}$$

or

$$ps = \beta y. \tag{4.6.6}$$

Therefore, the fraction is equal to a constant

$$x = \beta. \tag{4.6.7}$$

This result is identical to that for a Cobb-Douglas utility function without an externality. The network externality does not affect the demand function, only the utility level. Since the agents are not able to maximise their utility independently and according to the postulates of neoclassical economics, the resulting allocation will be inefficient. The interaction between the agents therefore has a market and a non-market component.

The non-market interaction can be translated to the market interaction. As a result, the value of the externality will be *internalised* in the market price. This idea represents the traditional neoclassical approach to external effects, by attempting to restore the Pareto efficient allocation, usually translated to a policy instrument—often a tax—to be used by an intervening government. Exploiting the symmetry in this example, the amount the other agent consumes can thought of as identical to the consumption of the first: $s_i = s_j = s$. Hence, the fraction of the income can be substituted in the utility function:

$$u = s^{\beta} z^{1-\beta} \left(\frac{ps}{y}\right)^{\gamma} = s^{\beta+\gamma} \left(y - ps\right)^{1-\beta} \left(\frac{p}{y}\right)^{\gamma}. \tag{4.6.8}$$

Optimisation implies

$$\begin{aligned}\frac{du}{ds} &= \left(\beta + \gamma\right) s^{\beta+\gamma-1} \left(y - ps\right)^{1-\beta} \left(\frac{p}{y}\right)^{\gamma} - p \left(1 - \beta\right) s^{\beta+\gamma} \left(y - ps\right)^{-\beta} \left(\frac{p}{y}\right)^{\gamma} \\ &= s^{\beta+\gamma-1} \left(y - ps\right)^{-\beta} \left(\frac{p}{y}\right)^{\gamma} \left[\left(\beta + \gamma\right)\left(y - ps\right) - \left(1 - \beta\right) ps\right] = 0. \end{aligned} \tag{4.6.9}$$

Solving this first-order condition results in

$$\left(\beta + \gamma\right)\left(y - ps\right) - \left(1 - \beta\right) ps = \left(\beta + \gamma\right) y - \left(\gamma + 1\right) ps = 0. \tag{4.6.10}$$

And finally

$$ps = \frac{(\beta + \gamma)}{(\gamma + 1)} y. \tag{4.6.11}$$

Or

$$x = \frac{(\beta + \gamma)}{(1 + \gamma)}. \tag{4.6.12}$$

Since $\beta < 1$, it follows that for $\gamma > 0$

$$\beta < \frac{(\beta + \gamma)}{(1 + \gamma)}. \tag{4.6.13}$$

This result shows that for the allocation to be efficient, for the same price the demanded quantity per agent, s, should be higher. Alternatively, for the same amount per agent, s, the agents ought to be willing to pay a lower price, in order to secure an efficient allocation. In terms of the benchmark exchange economy problem presented in section 4.2.3 the equilibrium quantities are known in advance to be $s^* = \frac{1}{2}A$ and $z^* = \frac{1}{2}$, since the total amount for both goods will be split equally between the two agents, again due to the symmetry assumption. Hence, the positive effect from internalisation on the (indirect) utility level results from a higher value for the externality, according to the substitution of both values of (4.6.13) in (4.6.1). Therefore, in the presence of network externalities, a Pareto improvement is possible in theory.

Although this result conforms entirely to the neoclassical concept of an external effect, the normative implications are likely to be limited in real policy applications. In terms of neoclassical welfare economics, the first agent benefits from the consumption of the second, and vice versa. They experience a contribution to their level of well-being that can not be directly related to the consumption of the good itself. In principle, the agents pay a price that is too high for the good to be allocated efficiently, compared to the price they should pay per quantity according to the utility they receive from consumption only. However, it might be argued that the additional contribution to the level of well-being from the consumption by other agents of the same type of good is an *endogenous quality characteristic* of the good as will be argued in section 4.6.3.

4.6.2 Product differentiation and exogenous quality

As a prelude to chapter 6, a model similar to the one in section 4.3 will be developed that focuses on a market equilibrium for *differentiated* goods. Before, the agent made a decision on how to divide her income between land and all other goods. Here, it will be assumed that land is available in two different varieties. A common method is the approach used by Dixit and Stiglitz (1977) which will be introduced here in a simplified way. Starting with an ordinary Cobb-Douglas utility

$$u(s, z) = s^{\alpha} z^{1-\alpha}, \tag{4.6.14}$$

the assumption that two varieties exist for good s is implemented through *nesting* of a second Cobb-Douglas utility for good s:

$$s = s_1^{\beta} s_2^{1-\beta}. \tag{4.6.15}$$

As a result, the utility for one agent is now a function of three goods:

$$u\left(s_1, s_2, z\right) = s_1^{\alpha\beta} s_2^{\alpha(1-\beta)} z^{1-\alpha}. \tag{4.6.16}$$

Similar to the indifference curves in section 4.2, an indifference *surface* can be distinguished in a three-dimensional plot. For a given utility level s, the amount of the numeŕaire, z, can be written as a function of the two varieties of s, s_1 and s_2:

$$z\left(s_1, s_2; u\right) = \left[\frac{s_1^{\alpha\beta} s_2^{\alpha(1-\beta)}}{u}\right]^{\frac{1}{\alpha-1}}. \tag{4.6.17}$$

Convexity now concerns a volume rather than an area as illustrated in figure 4.17. The Cobb-Douglas specification primarily has the benefit that the income is divided in fixed proportions over the goods. In this example, the amount of money spent on all other goods is $z = (1-\alpha)y$. It is therefore possible to separate the second decision—on how to divide the remaining αy over s_1 and s_2—from the first decision. This can be illustrated graphically by adding a surface to the original plot, as in figure 4.18. The intersection of this surface and the indifference surface can be considered an indifference curve for the two varieties of s. This approach is especially appealing if the Cobb-Douglas function of the second decision is replaced by a more general CES function[9]. In addition, an alternative specification for this function will be used. Instead of using coefficients β and $(1-\beta)$ that depend on the individual preferences, it will be assumed that these coefficients reflect a *quality level* that can be observed

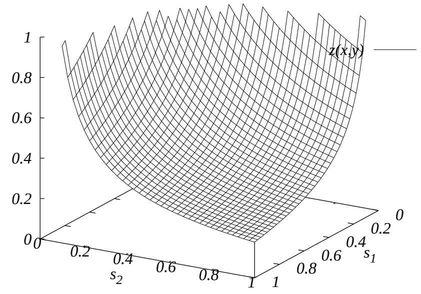

Figure 4.17: *Convex volume in case of two varieties.*

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[9] Recall that a Cobb-Douglas function has a CES = 1.0.

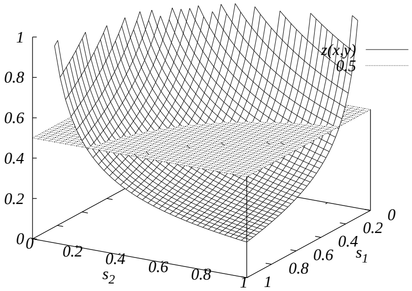

Figure 4.18: *Convex volume with intersecting plane.*

and that is independent from perception. The sub-utility function can be written as

$$\begin{aligned} u(s_1, s_2) &= \left(a_1^\rho s_1^\rho + a_2^\rho s_2^\rho\right)^{1/\rho} \\ &= \left[(a_1 s_1)^\rho + (a_2 s_2)^\rho\right]^{1/\rho}. \end{aligned} \tag{4.6.18}$$

Using the same procedure as in section 4.3, this CES function can be translated to a decision rule on what part of the remaining income αy on either s_1 or s_2. Based on an adaptation of (4.3.20), this decision concerns the utility levels

$$\ln v_{s_1} = \ln(\alpha y) + \ln a_1 - \ln p_{s_1}, \tag{4.6.19a}$$

$$\ln v_{s_2} = \ln(\alpha y) + \ln a_2 - \ln p_{s_2}. \tag{4.6.19b}$$

These equations can be inserted in a discrete choice model—or an equivalent decision rule—as well:

$$\begin{aligned} &\text{if} \quad \left(\frac{a_1}{a_2} > \frac{p_{s_1}}{p_{s_2}}\right) \quad \text{then} \quad x = \alpha, \\ &\text{if} \quad \left(\frac{a_1}{a_2} \leqslant \frac{p_{s_1}}{p_{s_2}}\right) \quad \text{then} \quad x = 0. \end{aligned} \tag{4.6.20}$$

This choice, based on the trade-off between quality and price, can also be implemented in a bargaining model in the same way as the original model in section 4.5. The combination with the first decision, on what part of his income the agent prefers to spend on the differentiated good, relative to the numéraire, will be discussed in more detail in chapter 6.

4.6.3 Product differentiation and endogenous quality

The final discussion concerns the combination of product differentiation and network externalities. It will be presented here in a rather stylised way, to stress the relation with the coordination game discussed in section 3.5. Introducing network externalities in a model with product differentiation seems more intuitive than the integration in the basic model, as in 4.6.1. Particularly the choice between similar products might be guided by the number people that bought the same type, brand or colour. With $a_1 = x^\gamma$ and $a_2 = (1-x)^\gamma$ a network externality can be introduced in (4.6.19), as

$$\ln v_{s_1} = \ln(\alpha y) - \ln p_{s_1} + \gamma \ln x, \tag{4.6.21a}$$

$$\ln v_{s_2} = \ln(\alpha y) - \ln p_{s_2} + \gamma \ln(1-x)\,. \tag{4.6.21b}$$

Assuming again that the two agents are identical, market clearing requires $p_{s_1}A = 2x\alpha y$ and $p_{s_2}A = 2(1-x)\alpha y$. As a result, a stylised 'best response', based on the network externalities only, can be written as

$$x = \frac{[x/(1-x)]^{(\gamma-1)/\mu}}{[x/(1-x)]^{(\gamma-1)/\mu} + 1}. \tag{4.6.22}$$

The right-hand side of (4.6.22) is plotted in figure 4.19 for $\gamma < 1$. The 'weight' the agent attaches to the network externality is a bifurcation parameter. If the weight of

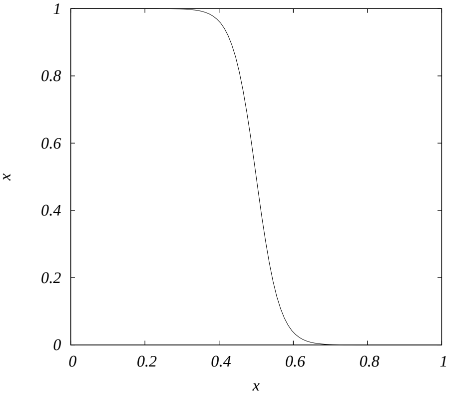

Figure 4.19: *'Best response' cf.* (4.6.22) *with* $\gamma = 0.2$ ($\gamma < 1$).

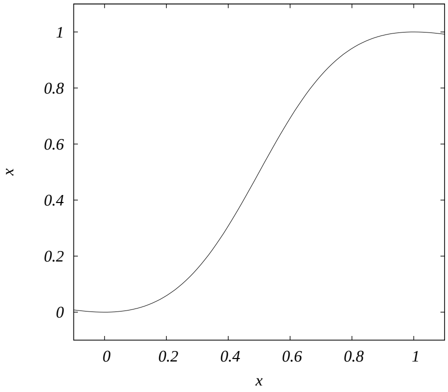

Figure 4.20: *'Best response' cf.* (4.6.22) *with* $\gamma = 1.2$ *(*$\gamma > 1$*).*

the network externality is smaller that the weight of the price (equal to 1 in (4.6.21)), the equilibrium $x = 1/2$ is unique and stable. In principle, (4.6.22) would give rise to multiple equilibria, $x = 1$ and $x = 0$, as in a coordination game for $\gamma > 1$, illustrated by figure 4.20. This implies that in the presence of network externalities only, and without quality differences between—or personal preferences for one of—the two varieties, both agents will choose only one of the two varieties[10].

4.7 Conclusions

This chapter discussed the problematic interpretation of the First Welfare Theorem. It is appealing as a normative result of neoclassical microeconomics , but a process description is lacking. A two-agent pure exchange economy served as an illustration of this issue.

It was shown that behavioural assumptions underlying utility maximisation are reinterpreted in terms of a best response, using the relation between a CES utility function and the logit model from the discrete choice literature. Based on the discrete choice formulation, the best response can be interpreted as a simple rule-based decision, distorted by circumstances captured in a single stochastic term.

[10] This extreme case, however, poses a problem for this particular, simple model. Because of the logarithms in (4.6.21), the 'solutions', $x = 1$ and $x = 0$, cannot be handled. A slightly more complicated definition of the endogenous quality levels, for example $a_1 = (1 + x)^\gamma$ and $a_2 = (2 - x)^\gamma$, would solve this problem, while sustaining the qualitative result.

The rule-based interpretation has the benefit over traditional utility maximisation that the rational choice is reduced to a best response. It highlights the problematic position of market clearing in the neoclassical model. Two possible solutions to this issue are explored. First the presence of a myopic auctioneer was assumed, who quotes disequilibrium prices. With a two-agent model, the convergence to the equilibrium price was shown to be sensitive for chaotic behaviour of the price, for certain values of the CES.

The second solution concerns a bargaining process based on reinforcement learning in games. Learning in games can be considered part of the literature on evolutionary game theory. The bargaining model illustrates the possibility of arriving at equilibrium prices and the corresponding neoclassical optimal allocation by means of a market interaction between two agents using rule-based decisions and minimal information. Although the approach has a few limitations, it shows that a bargaining solution for emergent optimality is feasible in theory. The model will serve as a basis for a population model in chapter 6.

4.8 Appendix

A few themes regarding the double exponential—or Type I extreme value—distribution are collected here. They concern the value of the mean, the expected utility, and consistency with the logistic distribution. The derivations and proofs are adapted from Anderson et al. (1992, p. 58–62) and Train (2003, p. 78–79).

4.8.1 Euler-Mascheroni constant

One possible occurrence of the Euler-Mascheroni constant, γ, results from the following integral that has no analytical solution

$$\gamma = -\int\limits_0^\infty e^{-x} \ln(x) dx. \tag{4.8.1}$$

Using the transformation

$$\begin{aligned} x &= st \\ dx &= sdt, \end{aligned} \tag{4.8.2}$$

the constant (4.8.1) appears in

$$\begin{aligned}\gamma = -\int_0^\infty e^{-st}\ln(st)s dt &= -s\int_0^\infty e^{-st}\left[\ln(s)+\ln(t)\right]dt\\ &= -s\ln(s)\int_0^\infty e^{-st}dt - s\int_0^\infty e^{-st}\ln(t)dt\\ &= -\ln(s) - s\int_0^\infty e^{-st}\ln(t)dt. \qquad (4.8.3)\end{aligned}$$

Here, use was made of the *Laplace transform* of 1:

$$\int_0^\infty e^{-st}dt = -\frac{1}{s}e^{-st}\Big|_0^\infty = \frac{1}{s}. \qquad (4.8.4)$$

Finally, from (4.8.3) follows the Laplace transform of the natural logarithm, in which the Euler-Mascheroni constant appears again:

$$\int_0^\infty e^{-st}\ln(t)dt = -\frac{\ln(s)+\gamma}{s}. \qquad (4.8.5)$$

Expression (4.8.5) will be used in section 4.8.3.

The value of the Euler-Mascheroni can be approximated as $\gamma \approx 0.577215665$.

4.8.2 Type I extreme value

For the double exponential—or Type I extreme value—distribution, the distribution and density are given by

$$\begin{aligned}F(\varepsilon) &= e^{-e^{-\varepsilon}}\\ f(\varepsilon) &= e^{-\varepsilon}e^{-e^{-\varepsilon}}. \qquad (4.8.6)\end{aligned}$$

In many cases, it is convenient to augment these definitions with a term that ensures a value of 0 for the mean. By definition, the mean is given by

$$\overline{m} \equiv \int_{-\infty}^{\infty} xf(x)\,dx = \int_{-\infty}^{\infty} xe^{-x}e^{-e^{-x}}dx. \qquad (4.8.7)$$

Using the transformation

$$\begin{aligned} t &= e^{-x} \\ x &= -\ln t \\ dt &= -e^{x} dx, \end{aligned} \tag{4.8.8}$$

together with (4.8.1), expression (4.8.7) can be rewritten as

$$\begin{aligned} \overline{m} &= \int\limits_{\infty}^{0} e^{-t} \ln t dt \\ &= -\int\limits_{0}^{\infty} e^{-t} \ln t dt = \gamma. \end{aligned} \tag{4.8.9}$$

Since the mean is equal to the Euler-Mascheroni constant, shifting the distribution by this constant results in a distribution for ε with a mean equal to 0:

$$F(\varepsilon) = e^{-e^{-(\varepsilon+\gamma)}} = e^{-ke^{-\varepsilon}}, \tag{4.8.10}$$

with $k = e^{-\gamma}$. The effect of this shift is illustrated in figure 4.21. Instead of a maximum value for the density at $\varepsilon = 0$, the adjusted distribution has two parts of equal size on both sides of this point.

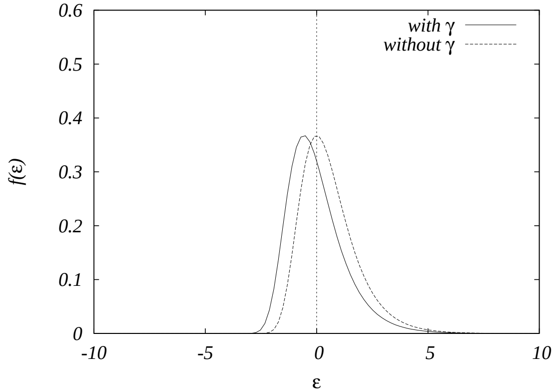

Figure 4.21: *Shift by γ to zero mean.*

4.8.3 Expected maximum utility

The choice can alternatively be interpreted as determining the maximum of the two values on either side of the inequality in the condition

$$\ln v_s + \mu\varepsilon_s > \ln v_z + \mu\varepsilon_z, \tag{4.8.11}$$

After dividing both sides of (4.8.11) by μ, the maximum value can be expressed as

$$\frac{\ln \tilde{v}}{\mu} = \max\left\{\frac{\ln v_s}{\mu} + \varepsilon_s, \frac{\ln v_z}{\mu} + \varepsilon_z\right\}. \tag{4.8.12}$$

However, due to the stochastic terms in (4.8.12), only the *expected value* of the maximum value can be determined:

$$\ln v = \mu\mathcal{E}\left[\frac{\ln \tilde{v}}{\mu}\right]. \tag{4.8.13}$$

Because the stochastic terms have a probability distribution according to (4.8.6), the distribution of the maximum value (4.8.13) can be written as

$$\begin{aligned} H(x) &= F\left[x - (\ln v_s)/\mu\right] F\left[x - (\ln v_z)/\mu\right] \\ &= e^{-ke^{-[x-(\ln v_s)/\mu]}} e^{-ke^{-[x-(\ln v_z)/\mu]}} \\ &= \exp\left\{-kLe^{-x}\right\}. \end{aligned} \tag{4.8.14}$$

Here,

$$L \equiv \left[e^{(\ln v_s)/\mu} + e^{(\ln v_s)/\mu}\right]. \tag{4.8.15}$$

The density that belongs to the distribution (4.8.14) is given by

$$h(x) = kLH(x)\,e^{-x}. \tag{4.8.16}$$

Using this density, the expected value (4.8.13) can be expressed as

$$\begin{aligned} \mathcal{E}\left[\frac{\ln \tilde{v}}{\mu}\right] &= \int_{-\infty}^{\infty} xh(x)\,dx \\ &= kL\int_{-\infty}^{\infty} xH(x)\,e^{-x}dx. \end{aligned} \tag{4.8.17}$$

With the transformation

$$\begin{aligned} t &= e^{-x} \\ x &= -\ln t \\ dt &= -e^{-x}dx, \end{aligned} \tag{4.8.18}$$

the expected value (4.8.13), using the definition (4.8.15), yields

$$\begin{aligned} \mathcal{E}\left[\frac{\ln\tilde{v}}{\mu}\right] &= kL\int_{\infty}^{0}\exp\left(-kLt\right)\ln t dt \\ &= -kL\int_{0}^{\infty}\exp\left(-kLt\right)\ln t dt \\ &= \gamma + \ln L - \gamma \\ &= \ln L \\ &= \ln\left[e^{(\ln v_s)/\mu} + e^{(\ln v_s)/\mu}\right]. \end{aligned} \tag{4.8.19}$$

Therefore, (4.8.15) can be stated as

$$\ln v = \mu\ln\left[e^{(\ln v_s)/\mu} + e^{(\ln v_s)/\mu}\right]. \tag{4.8.20}$$

And finally,

$$v = \left[v_s^{1/\mu} + v_z^{1/\mu}\right]^{\mu}. \tag{4.8.21}$$

4.8.4 Logit from double extreme distribution

For the evaluation of the condition

$$\ln v_s + \mu\varepsilon_s > \ln v_z + \mu\varepsilon_z, \tag{4.8.22}$$

the distribution (4.8.6), without adjustment for the Euler-Mascheroni constant γ, can be used because the deviation from a mean of 0 appears on both sides of (4.8.22). It can be rewritten as

$$\varepsilon_z < \varepsilon_s + (\ln v_s - \ln v_z)/\mu. \tag{4.8.23}$$

If ε_s is given, the fraction of the income spent on housing can be derived from the distribution of ε_z:

$$x_s|\varepsilon_s = F\left[\varepsilon_s + (\ln v_s - \ln v_z)/\mu\right]. \tag{4.8.24}$$

However, since ε_s is a random variable itself, the fraction needs to be derived as an expected value, according to

$$x_s = \int_{-\infty}^{\infty} F\left[\varepsilon_s + (\ln v_s - \ln v_z)/\mu\right] f\left(\varepsilon_s\right) d\varepsilon_s. \tag{4.8.25}$$

Solving this expression, involves

$$\begin{aligned} x_s &= \int_{-\infty}^{\infty} e^{-e^{-[\varepsilon_s + (\ln v_s - \ln v_z)/\mu]}} e^{-\varepsilon_s} e^{-e^{-\varepsilon_s}} d\varepsilon_s \\ &= \int_{-\infty}^{\infty} \exp\left[-e^{-[\varepsilon_s + (\ln v_s - \ln v_z)/\mu]} - e^{-\varepsilon_s}\right] e^{-\varepsilon_s} d\varepsilon_s \\ &= \int_{-\infty}^{\infty} \exp\left\{-e^{-\varepsilon_s}\left[e^{-(\ln v_s - \ln v_z)/\mu} + 1\right]\right\} e^{-\varepsilon_s} d\varepsilon_s. \end{aligned} \tag{4.8.26}$$

Using the transformation

$$\begin{aligned} t &\equiv e^{-\varepsilon_s} \\ dt &= -e^{-\varepsilon_s} d\varepsilon_s, \end{aligned} \tag{4.8.27}$$

it follows that

$$\begin{aligned} x_s &= -\int_{-\infty}^{0} \exp\left\{-t\left[e^{-(\ln v_s - \ln v_z)/\mu} + 1\right]\right\} dt \\ &= \int_{0}^{\infty} \exp\left\{-t\left[e^{-(\ln v_s - \ln v_z)/\mu} + 1\right]\right\} dt \\ &= \left. \frac{\exp\left\{-t\left[e^{-(\ln v_s - \ln v_z)/\mu} + 1\right]\right\}}{-\left[e^{-(\ln v_s - \ln v_z)/\mu} + 1\right]} \right|_{0}^{\infty} \\ &= \frac{1}{1 + e^{-(\ln v_s - \ln v_z)/\mu}} = \frac{e^{(\ln v_s - \ln v_z)/\mu}}{e^{(\ln v_s - \ln v_z)/\mu} + 1}. \end{aligned} \tag{4.8.28}$$

With regards to the expected value (4.8.19), it is interesting to note that the expected value appears as the denominator of (4.8.28):

$$\begin{aligned} x_s &= \frac{e^{(\ln v_s)/\mu}}{e^{(\ln v_s)/\mu} + e^{(\ln v_z)/\mu}} \\ &= \frac{v_s^{1/\mu}}{v_s^{1/\mu} + v_z^{1/\mu}}. \end{aligned} \tag{4.8.29}$$

For comparison, the probability distribution and cumulative density for the double exponential and logistic distribution are plotted together in figures 4.22 and 4.23.

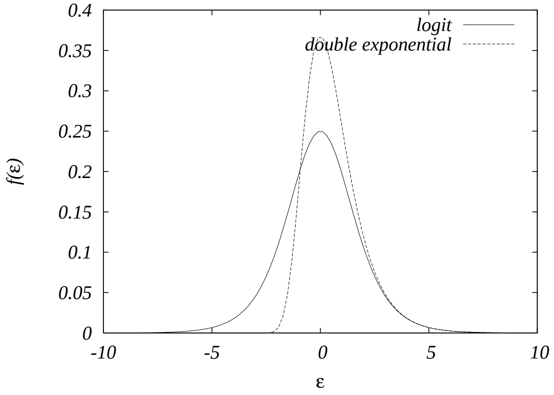

Figure 4.22: *Probability distribution for logistic and double exponential (*$\mu = 0.001$*).*

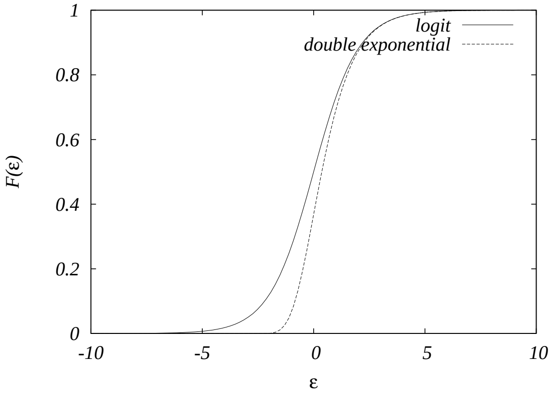

Figure 4.23: *Cumulative density for logistic and double exponential (*$\mu = 0.001$*).*

Chapter 5

Land use and welfare in economic theory

5.1 Introduction

Traditionally, the land market receives attention from various economic sub-disciplines. In each of these sub-disciplines the distinction between the land and the housing market is not always marked clearly. Land in urban economics is often considered a consumption good, with a corresponding competitive market. For simplicity this market is sometimes identified with the housing market, in which case it is typically referred to as the rental market for location space. In environmental economics land plays a role in property valuation, where the value of land is assessed in revealed preference methods, especially in *hedonic pricing*; again often projected on housing. In finance land is considered an asset—with extensions to real estate in general—, competing with other investments in diversified risk portfolios or as a basis for a credit loan (mortgage). These different interpretations highlight various aspects of land in economics. Leaving the asset, or *capital*, quality of land aside for the moment —primarily because of the complication of introducing time as yet another dimension, besides space—the question arises as to which combination of elements from spatial and environmental economics would serve policy interests concerning land markets from the perspective of public sector economics.

Public sector economics is concerned with the maximisation of the level of well-being—or (social) welfare—of all citizens in a given country, region or city. The initial observation in comparing welfare assessments in urban and environmental economics suggests that they accommodate different definitions of *welfare*. On one side, the urban economics literature stresses the optimal allocation of land through markets, while the environmental economics literature puts forward the public good character of local (environmental) *quality* on the other side. From a public policy point of view, an ideal welfare measure would address both aspects simultaneously. If, in a first assessment, the quality of a location would indeed be considered a local pure public good, exogenous to both consumer and producer (developer), two goods would play a role simultaneously on a land market:

1. *land*, as a consumer good or production factor, as in the urban economics land use tradition,
2. *quality*, as a local public good (amenity, environmental quality) in the tradition of environmental economics.

For this stylised case, public policy would be confronted with two aspects of a socially optimal allocation of land in the neoclassical framework:

1. securing optimal allocation of land by markets,
2. supplying local public goods.

If the quality is considered a characteristic of the land, both market and public good can in principle be combined in one welfare measure, as both goods are consumed simultaneously and contribute to the utility level of the individual agent. In this way, the normative aspects of land policy could indeed be assessed with an essentially neoclassical perspective. Land could in principle be allocated by a competitive market, while the public good interpretation of the quality implies that the characteristics of the land are fully exogenous for the consumer.

However, a complication arises in this case. The consumption of land can not be separated from the consumption of the local quality. The *local* public good property of local quality implies that only people at the location experience a direct benefit. As a result, the value of the quality affects the market price of land, due to the competition *between* locations. At least a part of the social value of the local quality is said to be *capitalised* in the land price. The capitalisation of the value of a public good is addressed frequently in the *public finance* literature. This especially concerns the theoretical underpinnings of taxes on real estate property. If a local government can influence the quality level because it is a public good, the local government will need to finance the supply of a quality improvement by public money. However, part of the benefit people experience from an increased supply of the local public good will be offset by a rise in the market rent and the land owner will therefore benefit, earning a higher income.

Chapter 1 stated that defining the goals of land policy becomes considerably more complicated if external effects are to be accounted for. In the neoclassical perspective, external effects result in an inefficient allocation of goods that is inefficient. The presence of positive external effects, however, needs to be assumed if the existence of agglomerations is explained by economic theory. Stated differently, only a framework different from the neoclassical general equilibrium model can explain why agglomerations are beneficial economically. The question of why agglomerations emerge has recently attracted most attention within *regional economics*, although it is addressed in urban economics as well. A possible explanation for the attention for agglomerations in regional economics lies in the use of models of *imperfect competition*. Regional economics is usually geographically explicit only up to the level of regions, while urban economics focuses on cities. Research in regional economics most often concerns the differences in development of indicators, such as the gross regional product and the unemployment rate, between regions. Many regional economists were already familiar with employing models of imperfect competition in international trade and the difference between countries and regions is primarily only an issue of scale. Topics such as the concentration of certain industries and competitive advantages can be studied at the level of continents, countries, or regions. If producers experience benefits in one region, this region might perform better—in economic terms—than another that lacks these benefits. If benefits depend on the presence of other producers they can be identified with agglomeration externalities. The literature that deals with agglomeration externalities in regional economics is often referred to as the *New Economic Geography* (NEG) (Krugman, 1991; Fujita et al., 1999).

As discussed in the previous chapters, imperfect competition together with game theory was the starting point for the adoption of complex systems methods in traditional economic theory in the 1990s. NEG models have many formal characteristics in common with models of social interactions. Examples are the *replicator dynamics* of evolutionary game theory in the NEG and the game theoretic interpretation of interaction-based methods in econometrics (Brock and Durlauf, 2001). The latter has already found its way into environmental economics. Recently, hedonic pricing methods have been extended by adopting location choice models for the valuation of non-marginal changes in levels of local amenities (Timmins, 2003; Smith et al., 2004; Bayer et al., 2005). These so-called *locational sorting models* are a good starting point for defining a single framework for assessing optimal land use and local quality. They adopt a type of location choice model that resembles the models in urban economics, while the interpretation as an extension of hedonic pricing reveals a connection with environmental economics. Furthermore, welfare assessments with locational sorting models focus on the capitalisation of the value of public goods in market prices for private property. This highlights a relation with the public finance literature, discussed above.

In this chapter, the several contributions to spatial economics will be discussed in more detail, grouped by the sub-disciplines of urban economics (5.2), public finance (5.3), environmental economics (5.5) and regional economics (5.4). Given the relevance in this thesis of locational sorting models as a bridge between traditional economics—combining elements from all sub-disciplines discussed in this chapter—and non-economic alternatives, they will be discussed separately in section 5.6. Conclusions will be drawn in section 5.7.

5.2 Urban economics

The origin of the urban economic tradition in land use models dates back to von Thünen (1826) for agricultural land use and its mathematical formalisation by Launhardt (1885). Von Thünen's method was extended by Alonso (1964), Muth (1968) and Mills (1972) for location choices of consumers and producers. These prototypes of urban economic models have always been interpreted as part of the neoclassical economic tradition, because they conform to the conditions for competitive markets. The market equilibrium price for land is assumed to be identical to the maximum *bid rent* of the individual agents. The bid rent represents the price a consumer or producer is willing to pay as rent after travel or transport costs are subtracted from her income. Travel or transport costs are the only connotation with geography, as they are calculated based on the distance to an exogenously given *Central Business District* (CBD), or market place. Traditional urban economics often assumes the existence of a continuous featureless plain, apart from the presence of a CBD.

5.2.1 Population approaches

In the urban economic land use models the bid rents—as market equilibrium prices—reflect a spatial equilibrium. The spatial equilibrium itself is defined in urban economics as an equilibrium *distribution* in densities of agents. Fujita (1989, p. 3–4) explains the extensive use of population densities in urban economics. In principle, a location choice represents a *discrete choice* for the individual household, as it will only buy or rent housing space of one sort at one location. Discrete choices violate the neoclassical assumption that all goods are divisible. Divisibility of goods means that a consumer can buy two goods in combinations of continuous quantities. If all land is considered one divisible good and if it is divided over a population of agents, the individual amount of land consumed per agent can be expressed as a *fraction* of the total amount of land. Equivalently, for any infinitesimally small piece of land, individual land use can be derived from the fraction of the population living at that location. It is assumed that the population size is large enough to permit the abstraction of a continuous population. The fraction of the population at a given location is therefore equal to the local population density. The distribution of agents is in this way similar to a *probability distribution,* with the local population density equal to the *probability* of finding an agent at a given location. This analogy will be exploited in the following chapters.

The population distribution in spatial equilibrium is Pareto optimal in the sense that no individual agent is able to improve her utility by moving to another location, without reducing the utility of other agents. For a population of identical agents, this definition of a spatial equilibrium implies that every agent—regardless of location—enjoys the same level of utility. Bid rents often implicitly assume a mechanism in which agents establish their bids in strategic interaction on the basis of equalising differences in utility. This mechanism suggests an analogy with the definition of utility similar to *game theory* and strategically price setting producers in an oligopoly. In urban economics in the Alonso tradition consumers would seem to be involved in strategic price setting by means of bid rents. This mechanism will be made explicit in chapter 6. The more traditional interpretation of the spatial equilibrium is identical to the common justification in neoclassical economics, relying on rational agents or an institution—the Walrasian Auctioneer—to set market clearing prices.

5.2.2 Limitations of the Von Thünen tradition

Although the prototype model sketched above of land use within or around a city can yield some insights in land use patterns, it is not capable of explaining the existence of the CBD itself. This shortcoming of the models in the Von Thünen tradition might be explained primarily by the relative dominant position of the neoclassical framework within economic theory. The models have also often been criticised, because of other unrealistic assumptions, such as the stylised representation of a city as a single line or a disk, but these assumptions are also grounded in the reference to the neoclassical framework. The main benefit of this reference is the possibility of extending the welfare theorems to the context of land use (Fujita and Thisse, 2002).

In the Arrow-Debreu model (Arrow and Debreu, 1954; Debreu, 1959), perfect competition is the main condition under which the Theorems of Welfare Economics can be applied. Perfect competition can be sustained in the traditional urban economics models, at the price of failing to give an economic explanation for the existence of cities. For agglomerations to be economically beneficial, product differentiation, specialisation and clustering need to be possible. Specialisation conflicts with the neoclassical assumption of the absence of scale economies, while clustering requires the presence of interactions in the shape of dependencies between utility and/or production functions; violations of the postulates cited on page 16 as well. The conclusion is that the emergence of cities cannot be explained under the neoclassical conditions for markets with perfect competition.

The existence of a CBD is occasionally justified by the assumption that a composite public good is supplied there. The composite good can be interpreted as a bundle of facilities a local government supplies or subsidises, for example counters for legal and administrative services at a town hall or museums and theatres. In principle, the distance to the CBD *itself*[1] could be considered an amenity (see also Scotchmer, 1986, p. 68, footnote 8). One additional benefit of interpreting the distance to the CBD as the characteristic of a location and replacing it by a local quality is the possibility to deviate from the rather restrictive linear relationship between quality and distance. As a result there is no strict necessity to represent a city with a one-dimensional line or a two-dimensional disk. Though, exogenous local quality levels allow for a neoclassical welfare analysis, they still fail to account for an endogenous CBD.

While models of increasing returns to scale have gained popularity, especially in regional economics, a few models in urban economics assume the existence of a production facility at the location of the CBD that operates with increasing returns (one example is Abdel-Rahman and Fujita, 1990). These models are typically interpreted as representations of ‘factory towns’ (Fujita and Thisse, 2002, p. 94) and reflect a

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[1] The shorter the distance to the CBD, the higher the local quality.

type of city that mainly emerged during the Industrialisation in the second half of the 19^{th} century.

Beckmann (1976) explains the existence of the CBD in a variant of the Alonso model as the result of *social interactions*. Instead of accounting for the cost of transportation to the CBD, transportation costs are based on the average distance to all other agents. Social interactions are therefore introduced as a highly stylised way of representing frequent visits. It implies that agents visit all other agents and that they do not combine visits in one trip. Nevertheless, the model illustrates how non-market interactions in combination with the traditional concept of a bid rent can give rise to emergent agglomerations. In a similar spirit, the assumption that the nearby presence of other producers has a positive impact on the production function of all producers might also account for the existence of a CBD, as shown by Lucas (2001) and Lucas and Rossi-Hansberg (2002). While the explanatory power of non-market effects might seem to be limited in theories on the emergence of agglomerations, they can play an important role as one element in a larger explanation, together with market effects. Stating that cities exist because people prefer to live close to each other does not reveal much. With social interactions as the main *centripetal* force and the price of housing as a *centrifugal* force, however, the structure of a city can be partly explained as the balance between the two. In the centre of the city, proximity to all other inhabitants is greater than in the suburbs. Because in principle people would prefer to live at a central place, housing prices in the centre are higher while the larger average distance to all other inhabitants in the suburbs is compensated for by lower housing prices. Furthermore, since housing prices are higher in the city centre, people demand less space there per person than closer to the city's border. As a result, the population density declines as the distance to the centre increases.

5.3 Public finance

Most analyses of the relation between the supply of local public goods and the impact on social welfare can be found in the public finance literature that takes the ideas of Tiebout (1956) as a starting point. Tiebout proposed to interpret a special kind of spatial equilibrium where a population is distributed over a given number of municipalities, as equivalent to a market equilibrium. The size of the population in a municipality would correspond to the demand for the locally supplied public good, determining in equilibrium its aggregate price as the municipal expenditure. Tiebout does not refer to capitalisation of the value of the public good in property prices directly, but Oates (1969) suggests testing Tiebout's hypothesis by examining differences in property tax, where this tax could be considered as the entry price of a municipality. According to Oates, property values must be higher in municipalities with a high level of public goods supplied. If people accept higher taxes for financing this supply, the net benefit for property owners has to be positive and Tiebout's conceptualisation would be correct.

5.3.1 Capitalisation

A part of the following research in this tradition focuses on estimating net benefits by measuring changes in rent (Lind, 1973). Starrett (1981) extends Lind's analysis of capitalisation. He also notices that capitalised rent counts as a benefit to the owner, but as a cost for the tenant. Although Starrett does not refer to Alonso (1964), his approach resembles the urban economic tradition of linking location choices to a trade-off between rent and travel cost, since the price a consumer pays for enjoying the public good can be related to expenses on both. As in Arnott and Stiglitz (1979), capitalisation can be linked to the Alonso model if it is assumed that the CBD itself is a public good, or the unique location where a public good is supplied. Because the local population density in the Alonso model is usually related to the inverse of space per person, the analysis of Scotchmer (1986) is especially worth mentioning in this respect. She makes a distinction between *short-run* and *long-run* benefits of environmental improvement. The difference between the two is that the long-run benefits take into account changes in the distribution of the population, or the population density. This can also be thought of as a difference between *partial* and *general* equilibrium analysis in terms of locational equilibrium models (Smith et al., 2004) and the related locational sorting models that will be discussed in section 5.6. The article by Scotchmer is also interesting because of its use of expenditure function in the welfare analysis. This makes her analysis conceptually related to the environmental economics tradition of Mäler (1974) for the valuation of non-market goods. Scotchmer, however, only relates the willingness to pay (WTP) for a marginal change in the location characteristics to an Alonso type of bid rent. This definition of WTP is therefore slightly different from the WTP usually applied in valuation methods, as will be discussed in section 7.2.

In the Tiebout model, the number of inhabitants of each municipality is endogenous. The spatial equilibrium in the sense of Tiebout, applied to several municipalities, would then imply the same level of welfare in every municipality. In urban economics the size of one city—in terms of total land use—is often assumed to be endogenous, while keeping the number of inhabitants fixed[2]. In both cases, the capitalised value of public goods supplied in a municipality or city is only reflected in the land price, or rent. Theoretically, it can be proven that the supplied public goods

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[2] A model of one city with fixed land use and an endogenous population size is referred to as an *open city* in urban economics literature.

could be financed completely by a property tax that equals the net rent; no further tax would be necessary (see for example Fujita and Thisse, 2002, p. 138). This neoclassical result is usually identified historically with the ideas of Henry George (Arnott and Stiglitz, 1979).

5.4 Environmental economics

The literature until recently showed little coverage of the spatial dimension in environmental economics, where land use would at first sight be an obvious research topic (Bockstael, 1996; van der Veen and Otter, 2001). One binding element for land prices in both environmental and spatial economics is the concept of a *bid rent*. In urban economics this concept plays a central role in land use modelling, as discussed in section 5.2. But housing prices have also been used extensively in environmental economics for a long time. In that respect, the perspective on land use in environmental economics seems to have been dominated by the theoretic underpinnings of hedonic pricing by Rosen (1974). Rosen proposed a perfectly competitive market for the characteristics of consumer goods making use of a bid rent concept, thereby referring to Alonso (Rosen, 1974, p. 38), but using the more conceptual ‘characteristics space’ rather than geographical space. Hedonic pricing has a special position in the context of methods applied in environmental economics in general—not restricted to land use. It is one of the few valuation methods with an explicit reference to market prices. Many other methods deal exclusively with the valuation of pure public goods. Starting with Mäler (1974), environmental economics has developed a theoretical basis for incorporating public goods and external effects—more generally, non-market goods—in an essentially neoclassical framework.

5.4.1 Valuation

In broad terms, non-market goods can be thought of as all goods that affect well-being, but which are not traded on a market. Environmental quality for example is assumed to be consumed, but there is no market for it. This allows for a clear separation in the analysis of the maximisation of welfare from the allocation mechanism. If people only consumed goods that are traded on markets, the price mechanism under perfect competition would secure an optimal allocation and thereby yield a maximum level of social well-being. If markets supplied environmental quality, it would typically lead to an under-supply, by familiar arguments that apply for public goods. From a welfare economics point of view, the state therefore is obliged to intervene in the allocation, either by regulation or by taking care of supply itself. When determining the optimal amount of public goods the government should supply to achieve a maximum level of social well-being, the government should ideally be able to estimate a demand curve in which the demand for quality is a function of a virtual price. Because public goods lack a market for achieving an efficient allocation, different criteria have been developed. Mäler (1974) proposed a concept of shadow or virtual prices, for public goods that is consistent with a standard definition of expenditure minimisation. The problem of expenditure minimisation is the dual problem of utility maximisation. It concerns the minimal income needed for achieving a given level of utility at given prices. The focus on expenditure minimisation in environmental economics can be explained by the goal of finding a monetary measure for welfare. To maintain the same level of welfare, while changing the supply of non-market goods, a minimal amount of virtual money that is needed can be accounted for as compensation in terms of income. If this compensation is negative, it is called a 'willingness to pay' (WTP). A positive compensation is a 'willingness to accept' (WTA). Both can be interpreted as a *monetary measure* for a change in the level of well-being, or welfare (Hanemann, 1999).

Valuation methods can be divided into revealed preference and stated preference methods. Revealed preference methods are based on the neoclassical behavioural model, with public goods included in the preference structure of the individual agent. For the consumption of the public good to be traceable in economic behaviour, a relation with a market-based choice must exist. Hedonic pricing is an example of a revealed preference method. When there is no such relation, a researcher must resort to creating an artificial market by means of a survey, where choices are recorded directly. The most often applied stated preference method is the *contingent valuation method* (CVM) (Hanemann, 1984). It can be regarded as the most rigorous implementation concerning the valuation of pure public goods if used for estimating the *existence value* of a public good. Both methods can play a role with respect to the land market. The contribution of the environmental quality of a parcel to the level of well-being of a consumer can be estimated with a revealed preference method, as it is assumed that will influence location choices. The value of a protected area might be estimated with a stated preference method, especially if people care about the presence of the area, even without using it actively for recreation. The preference structure assumed in CVM is also essentially neoclassical, for example in the assumption that the marginal utility of the public good decreases if the amount of the public good provided increases.

The demand for environmental quality, however, cannot immediately be compared with the demand for a market good since the derivation is different. In the case of a market good, a marginal change of its price has an effect on the demand for that good that can be understood as the combination of two first order effects. For a marginally *decreasing* price, these are

1. a marginal increase in the demanded quantity *while keeping the level of utility*

constant because the good has become more attractive than other goods as a result of a lower price; the *substitution effect*,

2. a marginal decrease in demanded quantity induced by the possibility of buying larger quantities of other goods because more income is available *when keeping the current demand for the first good at a constant level*; the *income effect,*

If both effects are taken into account, the demand function will represent a so-called *Marshallian* demand. If a *Hicksian* demand is to be derived, the second effect is omitted. For a normal market good both effects play a role and an ordinary demand for a market good is therefore a Marshallian demand. Because demand for environmental quality is derived from the minimisation of expenditure while keeping the level of utility constant in environmental economics, this demand concerns a Hicksian demand.

5.4.2 Price and quality

The introduction to this chapter argued that the value of land might be considered a combination of land as a market good and quality as a local public good. Quality can be interpreted as either a characteristic of a good or as a public good itself. Since quality is beyond the consumer's choice set, it is an exogenous factor and there is no formal basis for a distinction from a public good; the two can be regarded as synonymous. Some public goods are consumed only in combination with a market good. In that case the value of the public good can—in principle—be derived from empirical data on the consumption of the market good. This amounts to reconstructing a compensated (Hicksian) demand curve for the public good on the basis of an observed, uncompensated (Marshallian) demand curve for the private good. This problem was referred to above in section 5.4.1 and is relevant in the context of land markets.

One condition that allows for the reconstruction of a Hicksian consumer surplus (WTP) for a change in the level of the public good requires that the private good should be a *weak complement* to it public good (Haab and McConnell, 2002, p. 10). A complementary good is one that is only consumed together with some other good. 'Weakness' refers to the possibility of refraining from consumption of the complement. A theoretical quality level where the market price of the market good—as the weak complement—is so high that the demand for the weak complement goes to zero should exist. This is called the *choke price* and implies that the private good must be *non-essential*. If demand for the market good goes to zero, the demand for the public good reduces to zero as well. This is a condition for the possibility of estimating the value of an increase of the quality level. Increasing the quality level, as an exogenous impact on the level of well-being of the consumer, would otherwise always yield some benefit. Popular examples are found in the recreational use of the environment. A trip to the forest for example, involves the consumption of gas for the car, among other expenses. A family can decide to stay at home, thereby spending no money on gasoline for the forest trip. If the market price of gasoline rises, the family is less likely to go to the forest. The frequency will approximate zero if the price is so high that a trip to the forest is not important enough to use the car.

The exploitation of weak complementarity in the valuation of public goods involves various limitations, additional conditions and technical complications (Willig, 1978; Small and Rosen, 1981; Vartia, 1983; Bockstael and McConnell, 1993). The main difficulty is posed by the need to perform two translations consistently:

1. the welfare effect of a quality change needs to be translated to an equivalent price change, and

2. an observed change in Marshallian consumer surplus needs to be translated to a change in Hicksian consumer surplus.

The second translation is well-known (Willig, 1976), but an additional condition is needed in relation with the first, known as the *Willig condition* (Willig, 1978; Smith and Banzhaf, 2004). Roughly stated, the Willig condition maintains that the first translation should be independent of the income of the consumer. More precisely, the marginal rate of substitution between the quality and the price of the weak complement should be independent of income (Smith and Banzhaf, 2004).

It is important to note that the need for isolating a Hicksian surplus for quality changes is less relevant regarding land prices. Due to the effect of *capitalisation* discussed in section 5.3.1, it is rather a measure for the part of the value of quality changes that is not capitalised in the market price for land that is needed. This type of measure will be discussed in section 5.6. It is nevertheless appealing, to consider land prices in terms of weak complementarity. The environmental quality of a parcel of land could be interpreted as a locally available public good and as a weak complement to the land, which is—as a private good—traded on a market. An example of dealing with quality is represented by the *repackaging model*, usually ascribed to Fisher and Shell (1972). Their canonical example (Fisher and Shell, 1972, p. 26) concerns a box that initially contains ten widgets. If the same box—sold at the same price—contained twenty widgets, the ‘quality improvement’ of a box would equal half the price of the initial packaging. This example will be used in the models presented in the next chapters. The repackaging model conforms to the Willig Condition, but as indicated above, recovering the change in Hicksian consumer surplus is unnecessary if welfare effects from capitalisation are addressed.

5.4.3 Hedonic pricing

The method in which capitalisation is addressed explicitly is the well-established method of *hedonic pricing* (Rosen, 1974). Here a regression is performed on the price of a property as a function of various characteristics. When applied to housing, a change in the price of a house—due a change in some characteristics—could be interpreted as a monetary measure for the benefit of the change. If only the function for the price is available, the welfare analysis is usually restricted to marginal changes in the level of the public good. In line with the discussion above, the level of local environmental quality could be considered a characteristic. Based on the regression a marginal change in the quality level would be reflected in a marginal change of the price. The marginal change of the price can be considered the marginal value of the marginal quality change. Any analysis that goes beyond the valuation of marginal changes requires in addition to the specification of the price function a behavioural model from which a demand for the public good can be derived.

The behavioural underpinnings of hedonic pricing are supplied by Rosen (1974). Rosen (1974) defines the market price of the property good as a—possibly nonlinear—function of separate prices for all characteristics. In this way, hedonic pricing implicitly assumes that separate markets for characteristics exist. As far as these characteristics are determined by producers, Rosen (1974) is able to show that—under regular neoclassical conditions—hedonic pricing can be interpreted in terms of markets with perfect competition for characteristics. In a slightly different approach, Scotchmer (1986) derives an expression for a *bid rent* in hedonic pricing by adopting a reduced expenditure function, consisting of all expenditures except those for housing. The use of an expenditure function, with its implicit reference to compensation, makes her analysis conceptually related to the environmental economics tradition of Mäler (1974) for the valuation of non-market goods, discussed above. Analogous to the derivation of a Hicksian willingness to pay, a bid rent can be defined on the basis of the expenditure needed to attain (or maintain) a given level of utility. Based on this approach, a trade-off between WTP for the change in quality and a bid rent—as the market price in urban economics—is expected, when the utility level is kept fixed. While Rosen (1974) refers to a bid rent in character space, the bid rent Scotchmer (1986) applies can be identified directly with the one employed in traditional urban economics.

Scotchmer (1986) furthermore shows that the behavioural equations that are consistent with hedonic pricing can not be identified in general. This is essentially a negative result. If several behavioural assumptions are consistent with one and the same price function, there is no way of unambiguously deriving a demand function for the characteristics. More recent approaches in hedonic pricing follow a different approach. There, concepts from industrial organisation are adopted based on the analogy between product differentiation and discrete choice (Anderson et al., 1992; Berry et al., 1995). The general approach can be considered as a simultaneous estimation of demand and supply for differentiated goods. Especially interesting for the context of hedonic pricing is the fact that prices are endogenous in this model. It allows for the estimation of the value for non-marginal changes of characteristics. This feature will be discussed in more detail in section 5.6.

5.5 Regional economics

Recent advances in spatial economics—usually gathered under the heading *New Economic Geography* (NEG) (Krugman, 1991; Fujita et al., 1999)—show how models of imperfect competition might help explain the emergence of agglomerations in a general equilibrium framework. Due to its stringent assumptions, the traditional neoclassical general equilibrium model (Arrow and Debreu, 1954) is incapable of giving a spatially explicit representation of location choices of firms. Koopmans (1957) stresses the need for models that can handle *indivisibilities*. In section 5.2 the population approach used in urban economics was mentioned as a modelling solution for avoiding the need to take into account the indivisibility of location choices. This solution can be helpful if the main consumption decision concerns residential space, but regional economics focuses on the location where production takes place. Ordinary consumption goods will need to be considered indivisibles, because they need to be at least distinct in their location of production. In short, similar goods need to be differentiated. Goods can be differentiated by brand, colour, or other characteristics. It also has to be assumed that consumers are able to distinguish varieties and have a preference structure in which some varieties are preferred over others. This implies that goods cannot be perfect substitutes, even if they have a similar, if not identical use. Koopmans (1957) furthermore argues that a preference for variety on the side of the consumer, requires *increasing returns to scale* on the production side of the economy. According to the first postulate on page 16, increasing returns are in conflict with the neoclassical general equilibrium model.

With increasing returns to scale, for producers there exists the possibility to sell their products on a market for a price that lies above the marginal cost of production. Because the individual consumer is assumed to prefer a certain variety over others, there is a ‘price threshold’ before he will switch to another variety. In short, people are assumed to be willing to pay slightly more for their preferred brand than for a competing one. As a result, the producer of the preferred brand can benefit from scale economies.

If it is assumed that the marginal costs of production were constant, the total profit for the producer would increase with the amount of products sold. However, it is often more realistic to assume that making a product distinct requires an investment that reflects specialisation of the producer. In that case, the profit per product times the volume of production is needed to cover this initial investment. This is important for a more general discussion on welfare economics. A producer might need to charge a market price higher than the cost per product in order to be able to cover, for example, the costs of research and development. In strict neoclassical terms this would be inefficient, as the producer exploits increasing returns to scale. However, if the consumer is willing to pay the market price for an innovative product, rewarding the producer for her R&D investments, the 'inefficient' allocation does not imply a disadvantage for the consumer (see also Romer, 1990 and Grossman and Helpman, 1991).

With respect to regional economics, at least part of the preference for variety of the consumers can be used to cover transportation costs. Conceptually, it can be thought of as a minimal condition for export. In the standard Arrow-Debreu model transportation costs would be reduced to zero, effectively eliminating the notion of *distance*. As a consequence, a spatially explicit interpretation of the neoclassical framework would result implicitly in an autarky where, as the owners of the firms, consumers produce only for their own consumption.

5.5.1 Product differentiation and imperfect competition

In the most popular NEG model—the core-periphery model (Krugman, 1991; Fujita et al., 1999)—the Dixit-Stiglitz-Spence model (Dixit and Stiglitz, 1977; Spence, 1976) of *monopolistic competition* is assumed to be the only market type. This type resembles a market with perfect competition in the assumption that competition between producers forces them to accept a maximum profit that equals zero. It also resembles a market of oligopoly in the possibility to earn a net profit. Both assumptions can be reconciled by the additional assumption that the net profit is invested entirely in *fixed* costs of production. These fixed costs can be interpreted as the initial investment that is independent of the produces volume. It serves as a proxy for *specialisation*, as an investment in equipment needed for producing a good that can be distinguished from all other goods in the market. In principle less stylised variants of imperfect competition can be applied as well, such as Bertrand of Cournot oligopoly (Fujita et al., 1999, p. 52). In these market types producers are engaged in strategic price setting or strategic quantity setting, respectively. Strategic interaction between producers, though, would call for *game theory* to analyse the market equilibrium. This would complicate the analysis of the emergence of clusters. Monopolistic competition is sufficient for clustering, as it allows for *product differentiation* under minimal conditions. In this sense it is product differentiation rather than increasing returns that is at the heart of the NEG. However, product differentiation is not possible without increasing returns to scale. If producers can specialise, they can also account for transport costs. As a result, they can be located in a region other than the region where the goods they produce are consumed. In other words, by adding transport costs, the concept of import and export can be introduced in a general equilibrium model. Producers can choose the location that would maximise their profits, given the difference in volume of the goods they produce between the home market and the export markets. In the basic core-periphery model, locations are abstracted to two regions and therefore only one export market exists.

If the production side is characterised by product differentiation, the resulting imperfect competition will need to correspond to the preferences of the consumers. On the side of the consumers in the NEG, a utility function—also based on the Dixit-Stiglitz-Spence model—is assumed that reflects a preference for variety. If the producer specialises, his products will only be sold if consumers distinguish between the goods of various producers. In the Arrow-Debreu model, consumers consider the same products from different producers as *perfect substitutes*. They have a preference for certain goods and are indifferent to the producers that make them. However, people often have a favourite brand or shop. These preferences result in an elementary type of ‘niche’ for the producers. In the core-periphery model, these niches are just sufficient enough to generate a net profit for the producers that enables them to cover their fixed costs and transport costs. The preference for variety at the side of consumer is not without limitations, though. If the differences in prices between competing goods become too large, the consumer will opt for the good that was initially less preferred, because the difference in his utility level will be compensated by the lower price.

If the utility function of the consumer is interpreted as the utility function of a *representative* consumer, representing the average of an entire population, the single preference structure reflects a preference for differentiated goods. It contains a parameter that denotes the ‘level of differentiation’ indicating how different the representative consumer prefers the goods to be. There are two possible interpretations of this parameter. One interpretation considers the representative consumer as the ‘average consumer’ that prefers several goods that are similar, but different. In the second interpretation, the preference for variety would refer to the heterogeneity of the individual preferences within a population. Durable goods especially, are bought infrequently and an individual might have a strong *idiosyncratic* component in their preference structure for them. This component is made explicit in the interpretation of the multinomial logit model for a discrete choice as a model of product differentiation in Anderson et al. (1992).

5.5.2 Interactions and externalities

Once differentiation of products is allowed for in a modelling framework, it might in addition be assumed that the preferences of the consumers concerning product varieties are not entirely independent. Examples of this can easily be found in the real world, especially in the form of consumption externalities. The preference for a specific brand of clothes for example, might depend primarily on quality, but for many people it is also important whether the brand is fashionable or not. With a popular brand of clothes, part of the level of well-being to which wearing these clothes contributes depends on how many people are wearing clothes from the same brand. This is an example of a positive consumption externality, also known as *network externality*, discussed in chapter 4.

The core-periphery model of the NEG contains a more elaborate type of externality. In the model manufactured goods are differentiated while agricultural products are assumed to be perfect substitutes. The manufactured goods can be produced in two regions. The labour force for these types of goods is the only production factor and is assumed to be mobile. Workers, however, do not commute between regions. They are assumed to work in the region where they live. Because the labour force is the only production factor, the location choice of the workers as consumers also determines the location choice of the manufacturing industry, or the producers. Although in principle the number of varieties is endogenous, the specification of the model for monopolistic competition implies that in equilibrium the number is fixed. It is assumed that each variety is produced by a single firm and consequently, the number of firms is also fixed. With only two regions in total, the location choices in the model can be described by the *fraction* of firms located in one of the regions; the remaining fraction will be located in the other. Both manufactured and agricultural products from both regions can be consumed in either region, but bear transport costs only if produced in another region. For convenience, transport costs are defined by assuming that shipped goods can be considered ‘icebergs’, meaning that the costs can be expressed as the fraction of the volume that ‘melts’ during transport. This fraction links the consumption and production of the two regions and is thereby the basis for the external effects.

Positive externalities can arise in the region where most individual members of the labour force—and thereby the firms—choose to locate. The strength of the externalities depends on the transport costs, because the consumption in a region can be formalised as a mix between imported goods and goods produced in the own region. Firms prefer to be located in the region with the largest home market, because in that region the market prices bear no transport costs, market prices will be lower, and demand will be higher. Consumers prefer to locate in the region with the largest number of firms, because there they can earn a higher real wage. The real wage is the nominal wage divided by the *price index* in that region. Prices are lower in the region with products that are not imported. The firm can afford to pay a higher nominal wage there due to the economies of scale. The higher real wage for the consumers is a *forward linkage*, while the home market benefit for the producers represents a *backward linkage*. The combination results in a special type of positive feedback—or *self-reinforcing*—mechanism, similar to the coordination game in section 3.4. In the model, the dynamics of this mechanism are captured in an *evolutionary process*—formalised as the replicator dynamics, also discussed in chapter 3—that determines the rise in population density per region as a function of the difference in real wages between the two regions. In the case of relatively high transport costs, the export market will be small and the two regions will operate nearly independently. If transport costs decrease, the volume of exported produces will increase. As a result, the two regions become connected and—due to the external effects—firms will move to the region with the larger home market. The home market in that region will grow until all firms are located there, if the transport costs are low enough. In the core-periphery model, the transportation costs therefore serve as a bifurcation parameter. The bifurcation from one equilibrium to two equilibria can be identified with the bifurcation discussed in section 3.4.1.

Because the external effects in the core-periphery model operate through market prices—or, more specifically, in this case *transport costs*—, they are sometimes labelled *pecuniary externalities*. In this way, they can be distinguished from so-called *technological externalities* or *spillover effects*. The first are essentially *market interactions* while the latter are *non-market interactions*. This distinction is important with respect to the First Theorem of Welfare Economics. Allocation in the presence of market externalities is inefficient because of *imperfect competition*, while allocation in the presence of non-market interactions is inefficient due to welfare effects not internalised in market prices. In principle a market allocation in an imperfectly competitive market can be the best allocation that can be achieved, even though it is not socially optimal.

The distinction between the two types of externalities above is based on the perspective of the production side of the economy. The two types offer different possible explanations for the emergence of agglomerations in terms of the clustering of firms. Together with ‘thick markets for specialised skills’ (Fujita et al., 1999, p. 5), they comprise the three explanations for the formation of industrial clusters already identified by Marshall (1920). According to Marshall (1920) (Fujita et al., 1999, p. 4–5), clusters of firms might be explained by:

1. home market effects; both producers and consumers benefit from the absence of import costs,

2. labour market effects; firms prefer to be located in a region where a specialised

labour market already exists,

3. spill-over effects; firms directly benefit from the presence of other firms, because of communication, exchange of ideas, or inspiration by what is ‘around’.

The third explanation is intuitively clear, but is difficult to model without introducing assumptions that can effectively only be regarded as ‘black boxes’. In the literature they are sometimes referred to as *communication externalities*. Communication externalities can be thought of as *social interactions*, as discussed in section 5.2. Location choices of consumers are often determined by factors that contribute to their level of well-being, but are not allocated by markets; efficiently or inefficiently. Many people prefer to have neighbours, or at least they prefer to live in a community, ranging from a village to a mega-city. Some of the reasons can be identified with market interactions, such as the presence of shops, but non-market interactions are likely to play an important role as well.

A methodological issue arises regarding the presence of external effects—pecuniary or communication, and in general. The neoclassical framework is able to illustrate the Pareto-efficiency of an allocation governed by market prices, under well-specified conditions. If for some phenomenon—in this case agglomerations—these conditions cannot be met, it is not immediately imperative to resort to market interactions in an exploratory framework. If the neoclassical framework is to be abandoned, a multitude of alternatives emerge. As was argued in chapter 2, however, a model of human behaviour will still have to reside primarily on deductive logic since it cannot be based on universal laws. If the assumption of the existence of non-market interactions leads to the equivalent of a black box in certain explanations, it can still be useful in others. The citations below, illustrate the discourse in contemporary economic science. Glaeser (1999, p. 6) for example, writes in a plea for integrating non-market interactions in urban research:

> *‘Krugman (1991) shows that a brilliant theorist can explain cities without non-market interactions. But it is less obvious to me why one would want to do so. The flow of ideas and values that occurs through face-to-face interaction may be the most interesting feature of city.’*

On the other hand, Fujita et al. (1999) argue that explaining the emergence of agglomerations by the presence of increasing returns to scale can be particularly helpful if some elements in the environment change:

> *‘The larger point is that by modelling the sources of increasing returns to spatial concentration, we are able to learn something about how and when these returns may change—and then explore how the economy’s behaviour will change with them.’*

With a focus on the production side of the economy, this might be interpreted as a choice for keeping the economic analysis within the sphere of economic theory. A similar preference is expressed in Fujita and Thisse (2002, p. 303), explaining why market interactions are sometimes preferred over non-market interactions as the main element in the mechanism that results in an agglomeration:

> *‘Pecuniary externalities are also better studied within a general equilibrium framework to account for the interactions between the product and labour markets. Among other things, this allows one to study the dual role of individuals as workers and consumers.’*

Nevertheless, the role of non-market interactions, either as technological or communication externalities, also received attention in the recent economic literature. In this thesis, the modelling frameworks focus on residential land use and agglomerations will be assumed to emerge primarily as a result of non-market interactions. The role of non-market interactions will be explored especially in chapter 8.

5.6 Locational sorting

The research questions in chapter 1 concern the land market. The spatial economics literature in which this topic is addressed belongs mainly to sub-discipline of urban economics. As discussed in section 5.2, urban economic research traditionally devotes a substantial part of its attention to *land use* and cities. One way of accommodating insights from the use of models of imperfect competition and agglomeration externalities from regional economics in urban economics is the concept of *endogenous local quality* or *endogenous amenity* (Strong and Walsh, 2005). Endogenous local quality can be defined as a characteristic that is dependent on the location choices of other agents. It is an external effect, but at least part of its social value will be capitalised in the price of land. Although conceptually clear, the policy implications of addressing endogenous quality are difficult to account for. The main reason lies in the feedback mechanism that follows when it is assumed that households are mobile. The feedback mechanism this assumption introduces might be interpreted dynamically as follows:

- People choose a location based on price and quality,
- The location choices affect both price and quality of the locations,
- People revise their choices.

If quality is exogenous, the dynamics conform in principle to a regular process of finding a market equilibrium. Unfortunately, this mechanism is already not very well defined in the neoclassical model, as discussed in chapter 4. The mechanism, however, becomes really complicated if location choices, quality, and prices all have an impact on each other. Equilibrium selection in a model with endogenous quality conforms to a *complex dynamical system*. It contains both market interactions that can be shown to be consistent with the neoclassical urban economics models and might possibly be assessed in a self-organising system. It contains non-market interactions as well, which result in non-linearities in terms of interdependencies in the preference structures that deviated from the neoclassical linear activity model. Existing literature that deals with this type of non-market interactions and its relation with complexity usually adopts the term *social interactions*.

5.6.1 Social interactions and complexity

Koopmans (1957, p. 150–154) already sketched the outlines of the type of general equilibrium model that could eventually replace the Arrow-Debreu model if the clustering of economic activities had to be accounted for. The main obstacle according to Koopmans (1957, p. 154) referring to Koopmans and Beckmann (1957), is a formal one:

> *‘This is a situation ready-made for armchair theorists willing to make a search for mathematical tools appropriate to the problems indicated.’*

In economic theoretic terms, the mathematical tools refer primarily to *indivisible goods* and *increasing returns to scale*. The search called for by Koopmans might be identified with the development of tools that support the systematic investigation of the properties of nonlinear—or *complex*—dynamical systems. This not only applies to the role of increasing returns in the explanation of agglomeration formation. Elsewhere in his book in a discussion on *economic dynamics*, Koopmans mentions the ‘linearity of the behaviour equations’ (Koopmans, 1957, p. 179) as one of the ‘two main limitations of the tools used’ in the ‘last twenty years’(Koopmans, 1957, p. 179) at the time of his writing in 1957. The other limitation is the ‘highly aggregative character of the variables’(Koopmans, 1957, p. 179) in dynamic economic models.

The mathematical tools for handling indivisible goods and increasing returns are provided by the New Economic Geography (Krugman, 1991; Fujita et al., 1999). The NEG adopts the same selection mechanism as in evolutionary game theory—the *replicator dynamics*—, based on the fractions of a population choosing a region where the utility is above average, resulting in a Nash equilibrium. The utility in the core-periphery model can be interpreted as the real wage. The real wages in the regions are connected through transport costs.

Parallel to the development of NEG in the last decade a literature on social interactions has emerged, sometimes referred to as *New Social Economics* (NSE) (Becker and Murphy, 2000; Durlauf and Young, 2001). The modelling approaches adopted in this literature are similar in their mathematical structure to the core-periphery model of the NEG. One common feature is the reliance on techniques from game theory. Especially *evolutionary game theory* (Weibull, 1995) offers practical tools for equilibrium selection in the presence of interactions and bounded rationality (Young, 1998). The references to evolutionary game theory might also be interpreted as a way of applying theories from complex dynamical systems to economics (Durlauf and Young, 2001). The focus of the NSE, however, is different. While the NEG starts from the inherent difficulties of formulating a general equilibrium model that can account for emergent agglomeration, the NSE has a background in mathematical sociology rather than economics. It defines choices in a framework that resembles the work of Coleman (1990). But whereas the models by Coleman are frequently linear activity models in a spirit similar to neoclassical economics, the NSE stresses the complexity that might result from the non-linearities of certain types of social interactions. A second difference with mathematical sociology is the economic context to which many research problems refer to in terms of societal implications. Peer group pressure concerning alcohol abuse for example, might result in high costs for a society. Policy measures based on a traditional, neoclassical framework—usually a tax—intrinsically fail to address peer group effects, due to the behavioural assumptions that exclude social interactions. Following Brock and Durlauf (2001), a large part of the literature is devoted to the econometric identification of social interactions (Manski, 1995), using a discrete choice framework. Discrete choice models (McFadden, 1973, 1984) are common in econometric applications. The addition of interdependencies in the preference structures, though, results in characteristics of the models that can only be addressed using tools from the analysis of complex systems (Anderson and Arrow, 1988; Arthur et al., 1997; Blume and Durlauf, 2006). Finally, Brock and Durlauf (2001) even refer explicitly to Epstein and Axtell (1996) and also other linkages between social interaction models and agent-based modelling exist (Durlauf and Young, 2001).

5.6.2 Locational sorting and general equilibrium willingness to pay

The presence of social interactions can in principle be identified econometrically (Brock and Durlauf, 2001). This has inspired the development of new tools with applications in *environmental economics*, in support of property valuation (Palmquist, 2004; Bayer et al., 2002; Sieg et al., 2003). The derivation of a marginal WTP in hedonic pricing on the basis of the current price also implicitly assumes that the utility remains unchanged at the maximised level. If it is suggested that non-marginal changes in the local quality level might induce households to move to other locations, a demand function would needed to be specified so that new equilibrium prices could be derived together with a specification of supply. Because hedonic prices are considered market equilibrium prices where consumers maximise their utility level, equating demand and supply should always result in prices that remain consistent with utility maximisation. Stated differently, the calculation of endogenous equilibrium—market clearing, or hedonic—prices depends on the utility level in spatial equilibrium as an endogenous variable. The endogeneity of the utility level and more specifically, its dependency of the specification of supply and demand, characterise a special type of location choice models that has been developed for property valuation. These are the so-called *locational sorting models* (Bayer et al., 2002; Bayer and Timmins, 2002; Timmins, 2003).

Central in the welfare analyses of locational sorting models is the concept of a *general equilibrium to pay* (GE-WTP) (Smith et al., 2004). A GE-WTP should be able to account for the value of non-marginal changes in a spatially explicit distribution of local public goods, thereby extending current hedonic pricing methodology. Commonly, such a GE-WTP is derived as a Hicksian WTP adjusted for endogenous prices. Endogenous prices are typically enforced by a market clearing condition, often a fixed supply, constraining the relocation of a population in response to the changes in local quality. This strongly resembles the set-up of the basic urban economics models in the Alonso tradition. For a closed city model in urban economics, however, the city size is also endogenous. Therefore, a GE-WTP that allows for variation in the total amount of land used for residential purpose would be a suitable basis for further exploration of the combined welfare effects of quality and quantity aspects. This concept will be explored in chapter 7.

5.7 Conclusions

This chapter presented an overview is on the approach to land use within different sub-disciplines of economics. The main conclusion is that the *capitalisation* of the value of amenities in the price of land is the theme on which contributions from urban economics, public finance, and environmental economics can be united.

As far as spatially explicit economic issues are covered in mainstream economics, models that are part of the neoclassical tradition cannot account for the emergence of agglomerations. Recently, spatial economics has benefited from the applications of game theory and models of increasing returns. Increasing returns are implied by models of product differentiation in the New Economic Geography. Although these models are mainly adopted in regional economics—thereby lacking an assessment of land use—they offer formal concepts in which complexity has a clear meaning against the background of neoclassical economics in terms of external effects.

Locational sorting models integrate welfare notions from urban and environmental economics. The key issue of capitalisation is addressed in locational sorting models by means of endogenous prices following a specification of both demand and supply. Through their relation with the literature on social interactions, locational sorting models can be identified with the literature on complexity.

Chapter 6

An evolutionary interpretation of the Alonso model

6.1 Introduction

This chapter presents the complete derivation of the model employed in the next two chapters. The final model can be considered a real multi-agent system (MAS) and will be implemented as such in chapter 8. The starting point, however, will be the traditional Alonso model (Alonso, 1964). Although relatively restrictive in its applications, the original Alonso model is very flexible for hosting alternative interpretations and adding extensions. The main step in the model derivation concerns a reinterpretation of the original model as an *evolutionary* model. The basis for this reinterpretation is an analogy with a population game from evolutionary game theory discussed in chapter 3.

Furthermore, a stochastic variant of this population game allows for an additional reinterpretation of a spatial equilibrium. Based on the best response interpretation of CES utility function in chapter 4, the stochastic population game variant of the Alonso model can be shown to be consistent with a model of a *representative consumer*. This model provides a pragmatic general approach to the accommodation of a *population* in a welfare analysis. It can be interpreted as a simplified locational sorting model, while at the same time welfare effects are essentially captured in a quality-adjusted price index as the single indicator.

Based on the conclusions derived in chapter 5, the strategy of converting the original Alonso model can be summarised in three steps:

1. The distance to the CBD will be replaced by a local quality level,
2. The spatial equilibrium will be interpreted in terms of best response and Nash equilibrium for a population,
3. An evolutionary selection mechanism will be defined to explore the possibilities of ‘growing’ a city, by means of *self-organisation*, without resorting to an optimisation method that would lack a behavioural interpretation at the level of individuals.

Special attention will be devoted to welfare contribution of the city size. It will be shown that an endogenous city size allows for an integration of welfare notions from urban and environmental economics; the latter based on existing locational sorting models. This integration will be exploited in the welfare analyses in the next chapter.

This chapter is organised as follows. In section 6.2 a selection of elements is made that will be used as a characterisation of the land market. Section 6.3 revisits the Alonso model and develops a basic evolutionary equivalent of it. This equivalent will be generalised as a stochastic population game in 6.4, which in turn can be interpreted as a model of a representative consumer in a variant of the model in developed in chapter 4. A model with several populations will be used to demonstrate segregation by income—a general feature of existing sorting models—as a more advanced type of self-organisation in section 6.5. Section 6.6 closes this chapter with conclusions.

6.2 Basics of the land market model

To combine several welfare measures concerning land use planning in one modelling framework, three aspects will need to be addressed in advance. The first is the specification of the good that is being allocated. The characterisation of demand and supply on a market as a means for decentralised allocation is the second aspect; the third aspect concerns the formalisation of the location choices of individuals.

The main good allocated will be *space*, measured by its surface area. Space serves as an abstraction of land, because it allows for a separation of the demand for land from the location where the land is traded. Only land use by consumers will be taken into account. Neglecting producers in an economic assessment of land use decisions implies that the only land use type reflecting the presence of humans will be residential. It rules out the possibilities of conducting a welfare analysis that includes the effects of location decisions by firms. The introduction of firms would also require including the effects of wage formation, labour mobility, and other general equilibrium effects into the analysis. Although all these effects are important for an overall assessment of welfare effects in a spatially explicit context, they are likely to interact with—rather than determine—the welfare effects of land prices. Therefore, neglecting production allows for a better focus on the specific role public policy with respect to land use issues.

A market in which land is traded can in principle be characterised as part of a pure exchange economy. This is primarily a reflection of the fact that no land is being produced. Once owned, land can be considered an asset. Market prices for residential land use modelled in urban economics are often referred to as *rent*. A rent facilitates defining housing expenses for an individual or a household as a part of a budget for consumption of space in a repetitive time interval, monthly for example. This introduces the possibility that the consumer of the space is not by definition the owner of the piece of land. Land would be owned by means of initial endowments and can be rented from the owners by other consumers. The rent is the return on the asset for the owner. The main implication of assigning land property as an initial endowment in combination with a rental market is that land titles are assumed to be fixed. This assumption continues in all versions of the model developed. It is believed that the characterisation of the land market as a rental market in this way is sufficient for analysing the role of the government and its policy instruments for a possible regulation of the market. In terms of trade, there is no fundamental difference with a regular pure exchange economy if only one time period is considered because paying the rental price allows the tenant to consider the rented space his own for that period. If a tenant virtually rents the land he owns, the usual zero balance of income from endowments and expenses for consumables applies. Finally, considering rent as the return of the asset facilitates closing the model with respect to the land market in terms of 'general equilibrium', because land lords can be assigned explicitly and their welfare level can be analysed within the model if necessary. Interpreting the land market as a rental market also implies that the building sector will be neglected. The decision of whether or not to develop is taken by the owner or owners, of the land. It will be assumed that all land at a location—that is, the whole parcel—will be developed or not. Land is either used as residential space or left undeveloped as nature or agricultural land. Parcels can be owned by more than one individual and an individual can in principle own shares of several parcels. In urban economics it is often assumed that land owners are different from members of the population ('absentee landlords'). That assumption will be sustained in this chapter, but might later be relaxed, as will be discussed chapter 7.

From the separation of space from location it can be concluded that space is perceived as a differentiated good. It is differentiated in the first place by location and in the second by the location characteristics. A location choice will be interpreted as the choice for one of the available varieties of the 'product' space. As it will be assumed that each agent will only rent a certain amount of space of one variety at a time, the choice for the variety reflects his location choice. In the basic modelling framework developed in this chapter, a given total and finite amount of space is divided into M separate locations which can be imagined as parcels or 'grid cells'. Each grid cell either has a residential land use type, or no land use type. The latter can be interpreted as nature or in some cases also as agriculture. Following the tradition in urban economics, the grid cells will be distributed first on a one-dimensional line. Extensions to a two-dimensional grid will be introduced in chapter 8.

Evolutionary game theory (EGT) will be used as a reference point for modelling the location choices of individual agents in a population. EGT—and especially population games—offers a relatively simple approach to analysing the collective behaviour of one or more populations. A population consists in principle of a very large number of identical agents. Each agent chooses a single strategy out of a set of possible strategies. If choosing a strategy is identified with choosing a location, EGT offers a suitable basis for a location choice model. In a second stage, using techniques from agent-based modelling, agents can be modelled explicitly as individuals. This approach will be followed in chapter 8. The normative implication of this approach is the identification of spatial equilibrium with the defining equilibrium concept in evolutionary game theory: the Evolutionary Stable Strategy (ESS). The details were discussed in chapter 3, where it was noted that the ESS can be considered a refinement of the Nash equilibrium in classical game theory. Therefore, one issue to be addressed when adopting EGT as the basis for a location choice model is the relation between a Nash equilibrium in a population game and a market equilibrium on a land market. This will be done along similar lines as the two-agent model developed in chapter 4. Instead of a learning algorithm, a population approach will be adopted.

6.3 Alonso revisited

First, the traditional Alonso model (Alonso, 1964) will be adapted for an interpretation of differentiation by local quality and the relation with hedonic pricing discussed in chapter 5 will be highlighted. Fujita and Thisse (2002, p.79) derive the Alonso model starting with the following maximisation problem:

$$\max_{z,s} u(z,s) \qquad s.t. \qquad z + ps = y - t(r). \tag{6.3.1}$$

Here, y is income and p is the rental price per quantity, s, of *space*. The transportation costs t are a function of the distance, r, to the CBD. In a more general interpretation, r can be thought of as the coordinate of the location. The bid rent is defined as:

$$p(r, u^*) = \max_{z,s} \left\{ \frac{y - t(r) - z}{s} \qquad s.t \qquad u(z,s) = u^* \right\}. \tag{6.3.2}$$

This definition shows the relation between a bid rent function and an expenditure function which would be part of the dual problem of (6.3.1). It expresses the *willingness to pay* per quantity of space to attain a utility level of u^*. Because of the assumption that u is strictly convex, it is strictly increasing in z; z can be defined as a function (by inversion) of s and u^*. The bid rent can therefore be written as a function of s only, or

$$p(r, u^*) = \max_{s} \frac{y - t(r) - z(s; u^*)}{s}. \tag{6.3.3}$$

By definition of the bid rent the following equality holds (Fujita and Thisse, 2002, p. 80)

$$u^* \equiv v\left[p(r, u^*), y - t(r)\right]. \tag{6.3.4}$$

It expresses the fact that the demand that corresponds to the optimised bid rent is equivalent to the demand that would result from a traditional maximisation of utility. This is due to the maximisation of space, s, in (6.3.3) and the definition of the indirect utility. Assuming a homogeneous population, supported by the same problem (6.3.1) for every individual agent, a *spatial equilibrium* is defined by the same level of utility for every agent, u^*, regardless of her location, r. Market equilibrium prices in spatial equilibrium are therefore only a function of the location, r:

$$p^*\left(r\right) = p\left[y - t\left(r\right), u^*\right]. \tag{6.3.5}$$

For a given population size N, social welfare—which could here be identified with $U^* \equiv Nu^*$—can be interpreted as a function of the *city size*. The city size is measured in the maximum distance to the CBD, $r_{\max}$, where land is still occupied. This can be expressed as (Fujita and Thisse, 2002, p. 82)

$$p^*\left(r_{\max}\right) = p_A. \tag{6.3.6}$$

This results from the general condition $p^*\left(r\right) \geq p_A$, meaning that for the land owner the revenues from residential use need to be at least equal to rent earned from alternative (for example agricultural) use.

6.3.1 Bid Rent and local environmental quality

In addition to their use in urban economics, equilibrium prices are also commonly interpreted as bid rents in the hedonic pricing literature following Rosen (1974). Analogous to a willingness to pay, a bid rent can be defined on the basis of the expenditure needed to attain (or maintain) a given level of utility. Rosen proposed a perfectly competitive market for the characteristics of sites and houses, making use of the bid rent concept, thereby referring to Alonso (Rosen, 1974, p. 38). He defines a bid rent as a willingness to pay, according to:

$$u\left(y - ps; q\right) = u^*. \tag{6.3.7}$$

The parameter q is an exogenous quality index or public good[1] or *amenity*. As discussed in section 5.2 public goods seldom play an explicit role in the analysis in urban economics. Occasionally the existence of a CBD is justified by the assumption that a composite public good is supplied there. However, as suggested in Scotchmer (1986, p. 68, footnote 8), the distance to the CBD might be considered an amenity in itself. Quality would in that case depend on the location, r, and could be expressed as $q(r)$, representing the *local quality*. Therefore, in the following equations q will replace r in the original Alonso model. This is consistent with Rosen (1974, p. 34) referring to 'locational decisions in characteristics space', though this section follows Scotchmer (1986), rather than Rosen (1974). Scotchmer (1986) uses a reduced

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[1] Throughout this thesis the quality index, q, is assumed to be a scalar. This is done mainly to stress the similarity with the travel costs, t, in spatial economic models.

expenditure function in the derivation of the bid rent, consisting of all expenditures except those for housing. Her derivation can thereby easily be compared with Fujita and Thisse (2002) as referred to in (6.3.2). The maximisation problem then reads

$$\max_{z,s} u\left(z,s;q\right) \qquad s.t. \qquad z+ps=u. \tag{6.3.8}$$

The bid rent is defined as

$$p\left(q,u\right)=\max_{z,s}\left\{\frac{y-z}{s} \qquad s.t \qquad u\left(z,s;q\right)=u^{*}\right\}. \tag{6.3.9}$$

Again, with z strictly increasing in z, substitution by $z(s;q,u^{*})$ allows the bid rent to be written as a function of s only:

$$p\left(q,u^{*}\right)=\max_{s}\frac{y-z\left(s;q,u^{*}\right)}{s}. \tag{6.3.10}$$

For reference purposes, it is to be noted that the corresponding market equilibrium is defined by

$$u^{*}\equiv v\left[p^{*}\left(q\right),y;q\right]. \tag{6.3.11}$$

Expression (6.3.11) reiterates the congruency of the bid rent, the land price in market equilibrium, the hedonic price as a function of the local quality, and the market equilibrium as a *spatial equilibrium* in which the level of the (indirect) utility is independent of the location. Transportation costs are neglected in this analysis. In the original Alonso model, the transportation costs affect the indirect utility through an effect on the income. In the variant presented here, a similar effect is achieved by an immediate impact of the local quality level. It has the advantage that (6.3.11) already captures the intuition behind hedonic pricing in the interpretation followed in this thesis. Since the level of utility is independent of the quality, the land price apparently compensates for quality differences between locations.

6.3.2 Continuous Alonso model with a Cobb-Douglas utility

To illustrate the solution procedure in urban economics, the utility function will now be specified. As before, a special variant of the Alonso will be used in which utility is affected by a quality level, q, and transportation costs will be neglected. The utility of the individual household or agent will be represented by a Cobb-Douglas function

$$u\left(s,z;q\right)=s^{\beta}z^{1-\beta}q^{\gamma}. \tag{6.3.12}$$

The agent chooses how to divide his income between an amount of land, s, and all other consumables, z, while considering the local quality, q, as given. It is assumed that all income, y, will be spent. The price of land is denoted, p and the price of the consumption bundle is normalised to 1. This can alternatively be interpreted as taking money as the unit in which the bundle is expressed (see also section 4.2). The budget constraint is therefore given by

$$y = ps + z. \tag{6.3.13}$$

Maximising (6.3.12) with respect to (6.3.13) results in the *indirect* utility function

$$v\left(y, p; q\right) = \beta^{\beta}\left(1 - \beta\right)^{(1-\beta)} y p^{-\beta} q^{\gamma}. \tag{6.3.14}$$

This corresponds to a demand for land given by

$$s = \frac{\beta y}{p}. \tag{6.3.15}$$

Resorting to a *population* interpretation is common practise in urban economics (see section 5.2). It will also facilitate the translation to an evolutionary model in section 6.3.4. At a given location, r, the number of agents in equilibrium will be equal to n. Conforming to the original context, here r is interpreted as the distance to the exogenously given central business district (CBD). The total amount of land used at that location is equal to an infinitesimal small amount dr:

$$ns = dr. \tag{6.3.16}$$

Combining (6.3.15) with (6.3.16) yields

$$n = \frac{dr}{s} = \frac{p}{\beta y} dr. \tag{6.3.17}$$

If it assumed that the total number of agents, N, is fixed, this expression can be used in the integral that may be considered as an expression of the ‘conservation of agents’. Assuming a one-dimensional, symmetric city, the number of agents living between the CBD and city border, r_A, is given by

$$\int\limits_{0}^{r_A} \frac{n\left(r\right)}{dr} dr = \tfrac{1}{2} N. \tag{6.3.18}$$

Substitution of (6.3.17) in (6.3.18) results in

$$\frac{1}{\beta y} \int\limits_{0}^{r_A} p\left(r\right) dr = \tfrac{1}{2} N. \tag{6.3.19}$$

In urban economics, it is often presumed that a spatial equilibrium exists. This equilibrium is characterised by a level of utility that is equal for all agents if the preference structures of all agents are identical. The equilibrium utility level, u^* is furthermore assumed to be optimal in advance, adhering to the neoclassical origin from which the original model was derived. Actually optimality may be verified later[2]. The equilibrium utility level is therefore equal to the level of the indirect utility at all locations:

$$v\left(r\right)=u^*. \tag{6.3.20}$$

Next, the quality level, q, will be specified. To remain consistent with the original Alonso model, it will be assumed that the quality depends on the distance, r, to the CBD. This can be achieved for example by adopting the following definition of quality:

$$q\left(r\right)=\frac{1}{1+r}. \tag{6.3.21}$$

Although many other definitions are possible, (6.3.21) has the benefit of a straightforward interpretation of a steadily declining quality level as the distance to the CBD increases, while the quality level is exactly equal to 1 at the CBD.

Using the indirect utility function (6.3.14), the local land price, p, can be expressed as a function of the location, r:

$$p\left(r\right)=\alpha^*\left(1+r\right)^{-\frac{\gamma}{\beta}}. \tag{6.3.22}$$

Here, $\alpha^*=\left[\frac{1}{u^*}\beta^\beta\left(1-\beta\right)^{(1-\beta)}y\right]^{1/\beta}$. If the income, y, is given, the term α^* only contains the equilibrium utility level, u^* as an unknown. Substitution of (6.3.19) in (6.3.22) yields

$$\frac{\alpha^*}{\beta y}\int\limits_0^{r_A}\left(1+r\right)^{-\frac{\gamma}{\beta}}dr=\frac{\alpha^*}{\beta y}\left[\frac{\beta}{\beta-\gamma}\left(1+r\right)^{1-\frac{\gamma}{\beta}}\right]_0^{r_A}=\tfrac{1}{2}N. \tag{6.3.23}$$

Solving this expression for the location of the city border, r_A, results in

$$\left(1+r_A\right)^{\frac{\beta-\gamma}{\beta}}-1=\frac{\left(\beta-\gamma\right)y}{2\alpha^*}N. \tag{6.3.24}$$

The distance, r_A, between the CBD and the city border determines one half of the city size, measured in the number of agents. It is common in urban economics to

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[2] Verification of the (Pareto)-optimality of population distributions will be discussed in chapter 7.

assumed that city size measured in total area—here $2r_A$—depends on the *opportunity costs* of alternative land use. Usually this land use is identified with agriculture. Expressed as a condition, the owner of the land will require a price at least as high as the market price for the agricultural product that could be grown on the same land:

$$p\left(r\right) \geqslant p_A. \tag{6.3.25}$$

Since the equilibrium utility level, u^*, also applies to the residential land at the border of the city, the condition can be used for eliminating α^*. First

$$p\left(r_A\right) = p_A = \alpha^* \left(1 + r_A\right)^{-\frac{\gamma}{\beta}} . \tag{6.3.26}$$

Or

$$\alpha^* = p_A \left(1 + r_A\right)^{\frac{\gamma}{\beta}} . \tag{6.3.27}$$

Substitution of (6.3.27) in (6.3.24) results, after some manipulation, in

$$\left(1 + r_A\right) \left[1 - \left(1 + r_A\right)^{\frac{\gamma}{\beta}-1}\right] = \frac{\left(\beta - \gamma\right) y}{2p_A} N. \tag{6.3.28}$$

This expression must be solved for r_A numerically, for example by using a *bisection method* (Press et al., 2002).

With a numerical value for r_A, the entire model is solved. For later use, it will be convenient to normalise the number of agents per location, n/dr, to $2x/dr$ through division by $\frac{1}{2}N$. This allows for the interpretation of the integral

$$2 \int_0^{r_A} \frac{n\left(r\right)}{N dr} dr = 2 \int_0^{r_A} \frac{x\left(r\right)}{dr} dr = 1, \tag{6.3.29}$$

as an analogue to a *cumulative probability distribution*. An example of this population density distribution is given in figure 6.1 using the parameter values stated in table 6.1.

In line with Fujita and Thisse (2002), this variant of Alonso’s model in continuous space accounts for three stylised facts:

1. space per person increases as quality decreases,
2. population density decreases as quality decreases,
3. rent decreases as quality decreases.

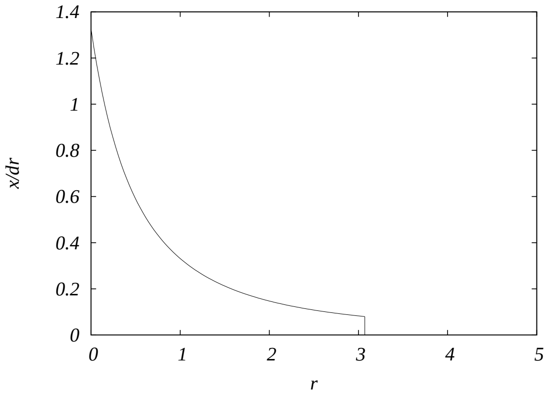

Figure 6.1: *Continuous case ($A = dr$).*

β	0.25
γ	0.5
y	1.0
N	10000
p_A	100.0

Table 6.1: *Parameter values for figures 6.1-6.3*

These stylised facts are valuable for the welfare analysis because they are in principle the result of an emergent market equilibrium. However, in addition to the simplifying assumptions concerning the homogeneity and the divisibility of the agents, the most problematic assumption is the existence of an equilibrium beforehand. The problem with this assumption will be shown to be identical to the problem of how market clearing prices are established in chapter 4. An evolutionary approach to market clearing in the Alonso model will be discussed in section 6.3.4, but first a discrete variant of the Alonso model is introduced.

6.3.3 A discrete Alonso model with a Cobb-Douglas utility

A first step toward the derivation of an evolutionary variant of the Alonso model consists of specifying a similar model with discrete locations of finite size, with an index j replacing the location coordinate r in the continuous model. Instead of dr, a location size of A is used, with

$$n_j s_j = A. \tag{6.3.30}$$

In principle, the location size can vary per location, but here it will be assumed for simplicity that all locations are of equal size. Instead of the integral (6.3.18) a summation is used by means of expressing the conservation of agents:

$$\sum_{j=1}^{M} n_j = \sum_{j=1}^{M} \frac{A}{s_j} = \frac{A}{\beta y} \sum_{j=1}^{M} p_j = \tfrac{1}{2} N. \tag{6.3.31}$$

Using the same utility function as before, an adjustment is also made for specifying the location by an index, j, instead of the distance r. As a result, the indirect utility at location j is expressed by

$$v_j = \beta^{\beta} \left(1-\beta\right)^{(1-\beta)} y p_j^{-\beta} q_j^{\gamma}. \tag{6.3.32}$$

Again, the equilibrium indirect utility level is independent of the location, hence $v_j = u^*$. The price at location j is given by

$$p_j = \alpha^* q_j^{\gamma/\beta}, \tag{6.3.33}$$

with $\alpha^* = \left[\frac{1}{u^*} \beta^{\beta} \left(1-\beta\right)^{(1-\beta)} y\right]^{1/\beta}$ as in (6.3.22). Substitution of (6.3.31) in (6.3.33), followed by a normalisation to a population density distribution results for the discrete case in

$$\frac{2\alpha^* A}{\beta N y} \sum_{j=1}^{M} q_j^{\gamma/\beta} = 1. \tag{6.3.34}$$

A comparison with

$$2 \sum_{j=1}^{M} \frac{n_j}{N} = 2 \sum_{j=1}^{M} x_j = 1, \tag{6.3.35}$$

reveals that

$$\sum_{j=1}^{M} x_j = \frac{\alpha^* A}{\beta N y} \sum_{j=1}^{M} q_j^{\gamma/\beta}. \tag{6.3.36}$$

This suggests that the discrete case has an analytical solution that can be written as

$$x_j = \frac{q_j^{\gamma/\beta}}{\sum_{k=1}^{M} q_k^{\gamma/\beta}}. \tag{6.3.37}$$

A more formal solution is presented in the appendix.

Finding the optimal city size for the discrete case is more elaborative than for the continuous case. The equilibrium city size is now expressed by the *number of locations*, M. The condition of the land owner in terms of opportunity costs remains the same:

$$p_j \geqslant p_A. \tag{6.3.38}$$

The location with the index $j = M$—the location at the city border—is the last location for which this condition holds. Therefore

$$p_M \geqslant p_A. \tag{6.3.39}$$

Using (6.3.33), the city's boundary condition can be written as

$$\frac{q_M^{\gamma/\beta}}{p_A} \geqslant \frac{1}{\alpha^*}. \tag{6.3.40}$$

From (6.3.34) it follows that

$$\frac{2A}{\beta Ny} \sum_{j=1}^{M} q_j^{\gamma/\beta} = \frac{1}{\alpha^*}. \tag{6.3.41}$$

Combining (6.3.40) with (6.3.41) results in

$$q_M^{\gamma/\beta} \geqslant \delta \sum_{j=1}^{M-1} q_j^{\gamma/\beta}, \tag{6.3.42}$$

with

$$\delta = \frac{\frac{2A}{\beta Ny}}{\frac{1}{p_A} - \frac{2A}{\beta Ny}} = \frac{1}{\frac{\beta Ny}{2p_A A} - 1} = \frac{p_A A}{\frac{1}{2}\beta Ny - p_A A}. \tag{6.3.43}$$

Expression (6.3.42) can be used in a simple recursive algorithm in which locations are added as long as the condition holds. This procedure implies the maximisation of the overall welfare level since $1/\alpha^* \sim (u^*)^{1/\beta}$. From (6.3.41) it follows that the value of $1/\alpha^*$ rises as the number of locations, M, increases.

Once the optimal number of locations is determined as described above, the population densities can be plotted. For reasons of consistency, the values for x_j/A will be used again, rather than the values of x_j. The reason for this lies in the possible confusion that might arise by interpreting the plot as a bar graph, while the width of

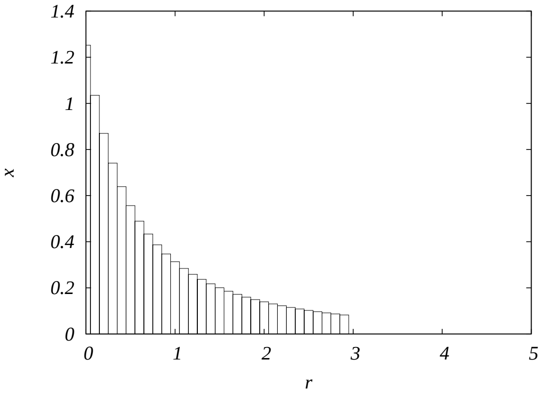

Figure 6.2: *Discrete case* ($A = 0.1$).

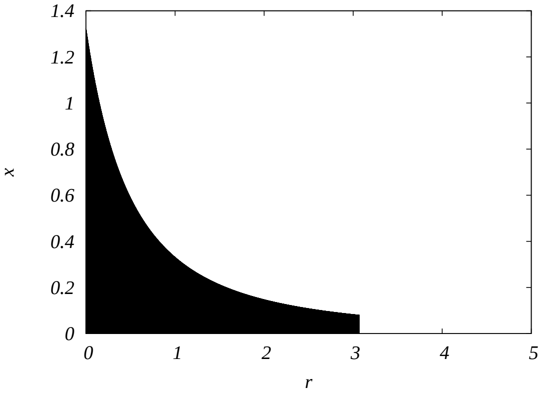

Figure 6.3: *Discrete approximation of figure 6.1* ($A = 0.001$).

the bar still represents the location size A. Therefore, the surface and not the height of each bar represents the number of agents per location. A first example is given in figure 6.2, for $A = 0.1$. Because an integration is basically the summation over the total area of the bars with an infinitesimal small width, the equivalence between the continuous and the discrete Alonso variant can be verified qualitatively by making a plot for $A = 0.001$ as in figure 6.3. It shows that this plot is virtually identical to figure 6.1 of the continuous model; the stylised facts of section 6.3.2 are also accounted for in this discrete variant of the Alonso model.

6.3.4 Location choices as evolutionary selection

The approach for finding the equilibrium distribution of agents in a discrete Alonso model as introduced in section 6.3.3 has several drawbacks. As in the continuous case, the main drawback is the lack of a description that can help explain the process of finding the equilibrium. Besides the problem of market clearing that also appeared in the discussion of standard neoclassical exchange model in chapter 4, the Alonso model also fails to explain how the equilibrium utility level emerges. One possible way of assessing the latter issue involves the translation of the discrete Alonso model to an evolutionary model. The detailed interpretation of the process at the individual level will be postponed until section 6.3.6. First, an interesting starting point will be highlighted by introducing the *replicator dynamics* (Weibull, 1995; Hofbauer and Sigmund, 1998) of evolutionary game theory (see also section 3.3):

$$\dot{x}_j = x_j \left(v_j - \overline{v} \right), \tag{6.3.44}$$

with $\overline{v} = \sum x_j v_j$. The dynamics result in an equilibrium solution if $\dot{x}_j = 0$. If $x_j > 0$ for all j, the equilibrium solution is achieved if

$$v_j = \overline{v}. \tag{6.3.45}$$

Stated differently, the equilibrium utility level, u^*, in section 6.3.3 can in this approach be considered identical to the *average* utility level, $\overline{v}$ in (6.3.44). As in chapter 4, it can be assumed that the demand for land at a location will be equal to the supply:

$$n_j s_j = \tfrac{1}{2} N x_j \frac{\beta y}{p_j} = A. \tag{6.3.46}$$

However, as long as the equilibrium of (6.3.45) is not reached demand will not be equal to the supply in the sense that the supply does not yet correspond to the optimal amount for every individual. In line with chapter 4, the *disequilibrium* land price will be assessed by using (6.3.46) for the total amount of money offered for the location, divided by the available amount of land. Rewriting (6.3.46), this price can be written as

$$p_j = \frac{1}{A}\frac{x_j \beta N y}{2}. \tag{6.3.47}$$

The total amount of money offered in (6.3.47) for location j is expressed as the fraction, x_j, of the total population on one side of the CBD, $N/2$, times the part, β, of the income, y, the individual agent spends on space.

Next, the opportunity costs for the land owner will need to be taken into account to determine the city border. This can be achieved as follows. Condition (6.3.38) might be interpreted as a *minimum price* the land owner requires to receive. The land owner will only accept the market price from residential use, if it is higher than the rent from agricultural use. This can be formalised as a final market price that is equal to the *maximum* of the prices from residential and agricultural rent:

$$\tilde{p}_j = \max\left\{p_j, p_A\right\}. \tag{6.3.48}$$

Inserting (6.3.47) in the indirect utility function, (6.3.32),

$$v_j = y\tilde{p}_j^{-\beta} q_j^{\gamma}, \tag{6.3.49}$$

closes the model. In (6.3.49) the constant terms $\beta^{\beta}\left(1-\beta\right)^{(1-\beta)}$ are left out for convenience, because the value of β is independent of the location.

The evolutionary dynamics that lead to the equilibrium distribution of agents is presented in figure 6.4. The plot shows the result of a numerical integration over time of the system of 50 coupled ordinary differential equations of the type (6.3.44), with the substitutions discussed above performed. Time runs from right to left to maintain the visibility of the stationary population density distribution that can be compared with figure 6.2.

The approach followed in this section illustrates how both the equilibrium utility level and the optimal city size can be made endogenous by introducing a simple evolutionary mechanism. Since the resulting equilibrium is identical to the equilibrium of the discrete variant of the Alonso model in section 6.3.3, the account of the stylised facts can be maintained. Here, they are given an emergent interpretation which allows for the interpretation of the Alonso model in terms of an elementary *self-organising system* in terms of a complex dynamical system, as discussed in chapter 3.

6.3.5 Spatial equilibrium as a Nash equilibrium

The concept of an evolutionary stable strategy (ESS) was introduced in the sections 3.4 and 3.5. Since the same replicator dynamics were used in section 6.3.4 as in

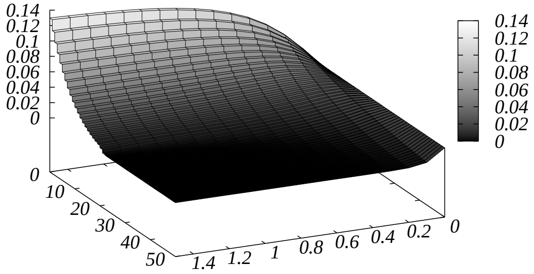

Figure 6.4: *Evolutionary dynamics for discrete Alonso variant.*

chapter 3, it can be concluded that the stationary state of the evolutionary discrete Alonso variant is a ESS. Since the ESS is a refinement of the Nash equilibrium, this implies that the spatial equilibrium can be interpreted as a Nash equilibrium in a population game. Because the location choices are considered strategies, this interpretation can be adopted in a very general sense, also without the Cobb-Douglas specification of the utility function or the disequilibrium price as in (6.3.47).

In principle any land use pattern can be thought of as the result of strategic interaction between all agents involved. If no agent has an incentive to move in a given configuration, the land use pattern apparently conforms to a ‘best response’ of the individual agent to the location choices of all other agents. At this level of abstraction, similar approaches can be found in Page (1999) and Otter et al. (2001). The more specific approach followed here defines strategic interaction within the context of the original Alonso model, while the equilibrium land use pattern can be identified explicitly with a Nash equilibrium in a population game. It thereby projects elements from alternative approaches to land use modelling back to traditional economic concepts, while adding the notion of self-organisation still. Furthermore, in section 6.4 the strategic interaction at the level of the population will be interpreted again in terms of fractions of the income spent, in line with the analysis of chapter 4, identifying the population with a representative consumer.

However, since the Alonso model is restricted to an exogenous CBD, the model in this chapter does not—contrary to Page (1999) and Otter et al. (2001)—, address the issue of the emergence of agglomerations. This issue will be postponed until chapter 8. First, another step will be taken. The replicator dynamics was introduced in this chapter without a behavioural justification. The justification will be presented next.

6.3.6 A behavioural interpretation of evolutionary selection

One possible way for the interpretation of the replicator dynamics in terms of the behaviour of individual agents is supplied by the model of *imitation*[3], adapted from Weibull (1995, p. 155–158) and Benaïm and Weibull (2003). Starting with a population of fixed size consisting of N nearly identical agents, the probability of one agent currently at location i changing his location is assumed to be a combination of two probabilities. The first is the probability of meeting an agent from location j. Given the assumption that N is very large, this probability might be considered identical to the population frequency at location j, x_j. The second probability is based on a comparison between the utility levels at both locations i and j, explained below. The probability of an agent switching from location i to location j will be denoted Pr_i^j and can—following the considerations above—be written as

$$\mathrm{Pr}_i^j = \begin{cases} x_j \cdot \mathrm{Pr}\left(v_j - v_i > -\varepsilon\right) & i \neq v \\ 1 - \sum\limits_{i \neq j}^{n} x_j \cdot \mathrm{Pr}\left(v_j - v_i > -\varepsilon\right) & i = j \end{cases}, \tag{6.3.50}$$

with $\varepsilon = \varepsilon_j - \varepsilon_i$. If it is assumed that ε has a *uniform* distribution, the probability density function is only defined over an interval $[a, b]$ (Ben-Akiva and Lerman, 1985):

$$f(x) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a < x < b \\ 0 & x > b \end{cases}. \tag{6.3.51}$$

The cumulative distribution yields

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x > b \end{cases}. \tag{6.3.52}$$

Within the relevant interval, $[a, b]$, this distribution can therefore be written as

$$F(x) = \frac{x-a}{b-a} = \alpha x + \beta. \tag{6.3.53}$$

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[3] For a derivation of the replicator dynamics closer to the original context of genetics, see the appendix of this chapter.

Here, $\alpha = 1/(b-a)$ and $\beta = a/(b-a)$. Using (6.3.53), the probability that depends on the comparison between the two utility levels can be written as

$$\Pr\left(v_j - v_i > -\varepsilon\right) = F\left(v_j - v_i\right) = \alpha\left(v_j - v_i\right) + \beta. \tag{6.3.54}$$

The comparison, $v_j - v_i > -\varepsilon$, in the first term of (6.3.54) might be interpreted as a decision rule for the individual agent, similar to the decision rule introduced in chapter 4.

Next, a balance between inflow and outflow can be constructed for every location i:

$$\begin{aligned}\dot{x}_i &= \sum_{j\neq i}^{n} x_j \mathrm{Pr}_j^i - \sum_{j\neq i}^{n} x_i \mathrm{Pr}_i^j \\ &= \sum_{j\neq i}^{n} x_j \mathrm{Pr}_j^i + x_i \mathrm{Pr}_i^i - \sum_{j\neq i}^{n} x_i \mathrm{Pr}_i^j - x_i \mathrm{Pr}_i^i \\ &= \sum_{j=1}^{n} x_j \mathrm{Pr}_j^i - \sum_{j=1}^{n} x_i \mathrm{Pr}_i^j = \sum_{j=1}^{n} x_j \mathrm{Pr}_j^i - x_i. \end{aligned} \tag{6.3.55}$$

Substitution of (6.3.54) in (6.3.55) results in

$$\begin{aligned}\dot{x}_i &= \sum_{j=1}^{n} x_j x_i F\left(v_i - v_j\right) - \sum_{j=1}^{n} x_i x_j F\left(v_j - v_i\right) \\ &= \sum_{j\neq i}^{n} x_j x_i F\left(v_i - v_j\right) + x_i x_i F\left(v_i - v_i\right) - \sum_{j\neq i}^{n} x_i x_j F\left(v_j - v_i\right) \\ &\quad - x_i x_i F\left(v_i - v_i\right) \\ &= \sum_{j=1}^{n} x_j x_i \left[\alpha\left(v_i - v_j\right) + \beta\right] - \sum_{j\neq 1}^{n} x_i x_j \left[\alpha\left(v_j - v_i\right) + \beta\right] \\ &= 2\alpha x_i \left[v_i - \sum_{j=1}^{n} x_j v_j\right] \\ &= 2\alpha x_i \left(v_i - \overline{v}\right). \end{aligned} \tag{6.3.56}$$

Apart from the coefficient 2α in (6.3.56), the final expression is identical to the replicator dynamics of (6.3.44).

The imitation model is shown to give a possible behavioural justification for the use of the replicator dynamics in section 6.3.5. The choices for both the mechanism of (6.3.50) and the uniform distribution for the stochastic term ε are rather arbitrary. However, they resemble similar elements in the procedure for deriving the equivalent decision rule for a CES utility function in chapter 4. Furthermore, in chapter 4 the market equilibrium was interpreted as a Nash equilibrium in fractions of the income. A close relation between the discrete variant of the traditional Alonso model and a standard two-agent exchange economy model will be shown to exist for logistic distribution for ε in section 6.4.

The relation between the ESS in this chapter and the Nash equilibrium in chapter 4 conforms to the standard interpretation of population games in evolutionary game theory. In classical game theory a mixed strategy is commonly interpreted as a randomisation of pure strategies to be played by a single agent. The probabilities are translated to fractions of the population playing pure strategies individually in a population game. In chapter 4, the mixed strategy was translated to the fraction of the income spent on land. From (6.3.47) it can be learnt that the same interpretation applies to the population model developed in this chapter. The total amount of money spent on space at location j was shown to be equal to the fraction of the total amount of money spent on land by the entire population. This amount was equal to $n_j \beta y = x_j \beta Y$, which is consistent with the model of product differentiation presented in section 4.7. The fixed fraction, β, of the income spent on land results from the Cobb-Douglas utility function. It might be generalised to a CES function, as will be discussed in section s644.

Finally, the mechanism applied in chapter 4 was inspired by the literature on learning in games. Learning can be interpreted as an evolutionary mechanism at the individual level. This also stresses the relation between the model developed in this chapter and the model presented in chapter 4 in terms of evolution and the selection of equilibria that allow for a normative interpretation. This interpretation will be applied in the welfare analyses in chapter 7. First the relation between the individual and the population model will be made fully consistent in the next section.

6.4 The Alonso model as a stochastic population game

The previous section noted that the introduction of the stochastic term in (6.3.50) is similar to the introduction of the stochastic term in the decision rule in chapter 4. Applying the same specification as in chapter 4 directly in an approach analogous to the derivation of the replicator dynamics in the previous section, results in an evolutionary dynamics related to it. Alternatively, this stochastic Alonso variant can be interpreted directly in a non-dynamic context as a reconciliation of the Alonso model and a logit choice model, as in Anas (1990). This section will show that the equilibrium solution of the resulting stochastic variant of the discrete Alonso model can also be interpreted as a model of a representative consumer with a preference for product variety similar to the use of the model by Dixit and Stiglitz (1977) in the New Economic Geography (Fujita et al., 1999). Finally, with an interpretation of the choice model for the representative consumer in terms of a nested logit model, the models of this section and of chapter 4 can be united for a CES utility function.

6.4.1 Dynamics

A description of the mechanics similar to (6.3.50) and essentially identical to the decision rule in section 4.3 can be written as (see Benaïm and Weibull, 2003 and Blume and Durlauf, 2003)

$$\mathrm{Pr}_i^j = \begin{cases} \mathrm{Pr}\left(v_j - v_i > -\varepsilon\right) & i \neq j \\ 1 - \sum\limits_{i \neq j}^{n} \mathrm{Pr}\left(v_j - v_i > -\varepsilon\right) & i = j \end{cases} . \tag{6.4.1}$$

With $\varepsilon = \varepsilon_j - \varepsilon_i$ from a *double exponential* distribution, the probability can be written as

$$\mathrm{Pr}\left(v_i - v_j > -\varepsilon\right) = F\left(v_i - v_j\right) = \frac{\exp\left(v_i/\mu\right)}{\sum_{j=1}^{n} \exp\left(v_j/\mu\right)}. \tag{6.4.2}$$

This implies a slight abuse of notation[4], because the multinomial logit (6.4.2) actually reflects a comparison of v_i with v_j from all other options, but it stresses the analogy with (6.3.50). Starting again from (6.3.55), but with (6.4.2) substituted re-

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[4] See Benaïm and Weibull (2003) and Blume and Durlauf (2003) for a more concise notation.

sults in

$$
\begin{aligned}
\dot{x}_i &= \sum_{j=1}^{n} x_j \mathrm{Pr}_j^i - x_i \\
&= \sum_{j \neq i}^{n} x_j \mathrm{Pr}_j^i + x_i \mathrm{Pr}_i^i - x_i \\
&= \sum_{j \neq i}^{n} x_j F\left(v_i - v_j\right) + x_i \left[1 - \sum_{j \neq i}^{n} F\left(v_i - v_j\right)\right] - x_i \\
&= \sum_{j \neq i}^{n} x_j F\left(v_i - v_j\right) - x_i \sum_{j \neq i}^{n} F\left(v_i - v_j\right) \\
&= \sum_{j \neq i}^{n} x_j F\left(v_i - v_j\right) + x_i F\left(v_i - v_i\right) - x_i \sum_{j \neq i}^{n} F\left(v_i - v_j\right) - x_i F\left(v_i - v_i\right) \\
&= \sum_{j=1}^{n} x_j F\left(v_i - v_j\right) - x_i \sum_{j=1}^{n} F\left(v_i - v_j\right) \\
&= \sum_{j=1}^{n} x_j F\left(v_i - v_j\right) - x_i \\
&= \frac{\exp\left(v_i/\mu\right)}{\sum_{j=1}^{n} \exp\left(v_j/\mu\right)} - x_i. \qquad (6.4.3)
\end{aligned}
$$

The main difference with the interpretation of the replicator dynamics in the previous subsection is the absence of imitation in the basic choice mechanism. In the derivation here it is implicitly assumed that the information on quality, disequilibrium price, and potential population density for all locations is publicly available. This assumption is rather strong, but it can be considered a starting point. Given this assumption, the dynamics can still be represented by a relatively simple set of ordinary differential equations.

As discussed above, the logistic distribution in the difference between error terms in the discrete choice variant suggests a relation with the CES function in chapter 4. This relation can now be finalised. First, in line with chapter 4, the dynamics for the *logarithm* of the indirect utility will be derived. Based on (6.4.1), the choice mechanism can be written as

$$
\mathrm{Pr}_i^j = \left\{ \begin{array}{c} \Pr\left(\ln v_j - \ln v_i > -\varepsilon\right) i \neq j \\ 1 - \sum\limits_{i \neq j}^{n} \Pr\left(\ln v_j - \ln v_i > -\varepsilon\right) i = j \end{array} \right. . \qquad (6.4.4)
$$

The transition probability resembles the expression for the fraction of the income of the individual spent on land in chapter 4:

$$\begin{aligned}\Pr\left(\ln v_i - \ln v_j > -\varepsilon\right) = F\left(\ln v_i - \ln v_j\right) &= \frac{\exp\left(\ln v_i/\mu\right)}{\sum_{j=1}^{M}\exp\left(\ln v_j/\mu\right)} \\ &= \frac{v_i^{1/\mu}}{\sum_{j=1}^{M} v_j^{1/\mu}}.\end{aligned} \tag{6.4.5}$$

Finally, after the substitution of (6.4.5) in the dynamics of (6.4.3) results in

$$\dot{x}_i = \frac{v_i^{1/\mu}}{\sum_{j=1}^{M} v_j^{1/\mu}} - x_i. \tag{6.4.6}$$

Next the indirect utility function (6.3.49),

$$v_j = y p_j^{-\beta} q_j^{\gamma}, \tag{6.4.7}$$

can be substituted in (6.4.6). Taking logarithms of (6.4.7) results in

$$\ln v_j = \ln y - \beta \ln p_j + \gamma \ln q_j. \tag{6.4.8}$$

Next an error term from a double exponential distribution can be added:

$$\ln v_j + \mu\varepsilon_{ij} = \ln y - \beta \ln p_j + \gamma \ln q_j + \mu\varepsilon_{ij}, \tag{6.4.9}$$

followed by a multiplication with $1/\mu$:

$$\frac{1}{\mu}\left(\ln v_j + \mu\varepsilon_{ij}\right) = \frac{1}{\mu}\ln y - \frac{\beta}{\mu}\ln\left(p_j\Big/q_j^{\gamma/\beta}\right) + \varepsilon_{ij}. \tag{6.4.10}$$

As a result

$$x_j = \frac{\left(p_j\Big/q_j^{\gamma/\beta}\right)^{-\beta/\mu}}{\sum_{k=1}^{M}\left(p_k\Big/q_k^{\gamma/\beta}\right)^{-\beta/\mu}}. \tag{6.4.11}$$

The dynamics (6.4.6) can be written as

$$\dot{x}_j = \frac{\left(p_j\Big/q_j^{\gamma/\beta}\right)^{-\beta/\mu}}{\sum_{k=1}^{M}\left(p_k\Big/q_k^{\gamma/\beta}\right)^{-\beta/\mu}} - x_j. \tag{6.4.12}$$

It can be shown that (6.4.11) converges to (6.3.37) if $\mu \downarrow 0$. It means that if the process reaches its stationary equilibrium—$\dot{x}_j = 0$ for all j—and the variance of the error term reduces to zero, the solution is identical to the equilibrium population distribution found in section 6.3.3. Substitution of $p_j = x_j \beta N y / A$ in (6.4.11) results in

$$x_j = \frac{\left(x_j \Big/ q_j^{\gamma/\beta} \right)^{-\beta/\mu}}{\sum_{k=1}^{M} \left(x_k \Big/ q_k^{\gamma/\beta} \right)^{-\beta/\mu}}. \tag{6.4.13}$$

Solving for x_j (see appendix) and taking the limit finally yields

$$\lim_{\mu \downarrow 0} x_j = \lim_{\mu \downarrow 0} \frac{q_j^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}} = \frac{q_j^{\gamma/\beta}}{\sum_{k=1}^{M} q_k^{\gamma/\beta}}. \tag{6.4.14}$$

This expression is identical to (6.3.37).

An alternative, non-dynamical, introduction of a stochastic variant of the Alonso model is presented in Anas (1990). The indirect utility function (6.4.9) can be thought of as a result of the maximisation of

$$u_{ij}\left(s_j, z; q_j, \varepsilon_{ij}\right) = s_j^{\beta} z^{1-\beta} q_j^{\gamma} e^{\varepsilon_{ij}}. \tag{6.4.15}$$

This approach is also followed in the locational sorting model of Timmins (2003).

6.4.2 Aggregation

The model developed in this section allows for direct aggregation of the welfare level for N agents. It facilitates the interpretation of the social welfare function as the indirect utility function for a *representative consumer*. The expected value of the logarithm of the individual indirect utility, maximised over all possible options j, is given by (see section 4.8, the appendix of 4)

$$\mathcal{E}\left(\max_{j=1\ldots M} \ln \tilde{v}_{ij}\right) = \mu \ln \left[\sum_{j=1}^{M} \exp\left(\frac{\ln v_j}{\mu} \right) \right]. \tag{6.4.16}$$

Since this expression is based on the distribution $F(x) = \Pr(\ln v_j + \varepsilon_{ij} \leq x)$, a monotonic transformation leaves the result unaffected, because

$$\Pr(\ln v_j + \varepsilon_{ij} \leq x) = \Pr(v_j e^{\varepsilon_{ij}} \leq e^{x}). \tag{6.4.17}$$

As a result, the expected maximum level of utility for the individual agent is given by

$$\mathcal{E}(\max_{j=1\ldots M} v_j e^{\varepsilon_{ij}}) = \left(\sum_{j=1}^{M} v_j^{\frac{1}{\mu}}\right)^{\mu}. \tag{6.4.18}$$

It follows that with a specification of the indirect utility function according to

$$v_j e^{\varepsilon_{ij}} = \beta^{\beta} (1-\beta)^{(1-\beta)} y p_j^{-\beta} q_j^{\gamma} e^{\varepsilon_{ij}}, \tag{6.4.19}$$

the social welfare function can be written as

$$N\mathcal{E}(\max_{j=1\ldots M} v_j e^{\varepsilon_{ij}}) = \beta^{\beta} (1-\beta)^{(1-\beta)} Y G^{-\beta}, \tag{6.4.20}$$

where $Y = ny$ and

$$G \equiv \left[\sum_{j=1}^{M} \left(\frac{q_j^{\gamma}}{p_j^{\beta}}\right)^{\mu}\right]^{-\frac{\mu}{\beta}} = \left[\sum_{j=1}^{M} \left(\frac{p_j}{q_j^{\frac{\gamma}{\beta}}}\right)^{-\frac{\beta}{\mu}}\right]^{-\frac{\mu}{\beta}}. \tag{6.4.21}$$

Expression (6.4.21) represents a prices index; its role will be discussed in more detail in section 6.4.3.

The aggregated value represented by (6.4.20) actually consists of a summation of the *expected* level of utility for all individuals. It is identical to the indirect utility function of the representative consumer, as applied in the basic core-periphery model in the New Economic Geography (Fujita et al., 1999), as will be shown in section 6.4.3. The expected value in (6.4.18) has a position similar to the level of utility in spatial equilibrium, u^*, in section 6.3.

This is more apparent in the comparison with (6.3.45), the average indirect utility level in the replicator dynamics. With the population densities interpreted as choice probabilities, the average indirect utility, $\overline{v} = \sum_{j=1}^{M} v_j$, is identical to the *expected* indirect utility level. However, due to the presence of the stochastic terms, the interpretation of (6.4.20) as a *social welfare function* is only valid with a very large population of agents that are identical up to the idiosyncratic term, ε_{ij}. Separate draws from the distribution for this term should be present in the preference structure for every agent, for every location. This restriction is one of the motivations of the translation of a similar model into an agent-based model or multi-agent system in chapter 8.

Substitution of (6.4.13) in (6.4.20) results in an analytical solution for the social welfare function:

$$V = A^{\beta} \left[(1-\beta)Y\right]^{1-\beta} \left(\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta+\mu}}\right)^{\beta+\mu}. \tag{6.4.22}$$

It follows that social welfare with endogenous prices is only a function of the quality of all sites and the number of sites, M. Assuming that the population relocates itself over the initial supply, M can be kept fixed and the impact of local changes in quality on social welfare can be examined. This function will be analysed further in chapter 7.

6.4.3 Representative consumer

The main benefit of adopting the dynamics of (6.4.12) and its relation with a CES utility function is the possibility of interpreting the location choices of N agents in terms of consumption by a single *representative consumer*. The relation between a CES utility function for a differentiated good and the multinomial logit model applied in this chapter follows therefore Anderson et al. (1992) closer than the special interpretation of this relation presented in chapter 4. The use of a representative consumer is often criticised because of its lack of realism. If a representative consumer represents N individuals, this can alternatively be interpreted as using N identical, average individuals, as shown in section 6.4.2. It can be argued that the problematic interpretation of the representative consumer in the model developed in this chapter mainly originates in the difficulties that arise when using the behavioural rules of the *average* agent. If the step of a translation to an evolutionary model is followed by a second translation in a *multi-agent system,* it can contribute to land use modelling within economics by means of systematic *disaggregation* of a population into *heterogeneous* individuals. This will be shown in chapter 8.

From the Cobb-Douglas utility (6.3.12) it follows that every agent in the population spends a fraction β of his income on housing. The total income of all agents is equal to $Y = Ny$. The representative consumer—representing the entire population—therefore spends βY on housing. Per location $x_j \beta N y = p_j A$ is spent, as noted in section 6.3.6. This is consistent with a Cobb-Douglas utility function for the representative consumer with a CES sub-utility function for land as a *differentiated* product. This function is similar to the use of the Dixit-Stiglitz (Dixit and Stiglitz, 1977) function in the New Economic Geography (Fujita et al., 1999). Its use has the benefit of capturing all the necessary information on spatial welfare in a single indicator.

The representative consumer has the following utility function:

$$U\left(S, Z\right) = \tilde{S}^{\beta} Z^{1-\beta}. \tag{6.4.23}$$

Here, $\tilde{S}$ is the total amount of space used, adjusted for quality, as explained below. It is the sum over M locations of size S_j, according to

$$S = \sum_{j=1}^{M} S_j. \tag{6.4.24}$$

As indicated, the income of the entire population will be interpreted as the income of the representative consumer. The amount of money spent on land is the sum over all locations with a price p_j per location:

$$Y = Z + \sum_{j=1}^{M} p_j S_j. \tag{6.4.25}$$

The total amount of *quality-adjusted* space will be defined as

$$\tilde{S} = \left[\sum_{j=1}^{M} \tilde{S}_j^{\rho} \right]^{1/\rho} = \left[\sum_{j=1}^{M} (a_j S_j)^{\rho} \right]^{1/\rho}. \tag{6.4.26}$$

Here,

$$a_j = q_j^{\gamma/\beta}, \tag{6.4.27}$$

for simplicity, since q, β and γ are all considered constants. Following Fujita et al. (1999, p. 46–47), the maximisation of (6.4.23) with respect to (6.4.25) will be performed in two steps. The consumption of space will be treated as equivalent to an expenditure minimisation problem where the expenditure on space will be minimised according to

$$\min \sum_{j=1}^{M} p_j S_j \quad s.t. \quad \tilde{S} = \left[\sum_{j=1}^{M} (a_j S_j)^{\rho} \right]^{1/\rho}. \tag{6.4.28}$$

The expenditure on space can be rewritten to bring it in line with the quality-adjusted amount of space in the constraint as

$$\sum_{j=1}^{M} p_j S_j = \sum_{j=1}^{M} (p_j / a_j)\,(a_j S_j). \tag{6.4.29}$$

From first order conditions, it follows that

$$\frac{(a_j S_j)^{\rho-1}}{(a_i S_i)^{\rho-1}} = \frac{(p_j / a_j)}{(p_i / a_i)}. \tag{6.4.30}$$

Substitution of (6.4.30) in (6.4.29) results in

$$a_i S_i = \frac{(p_i/a_i)^{1/(\rho-1)}}{\left[\sum_{j=1}^{M}(p_j/a_j)^{\rho/(\rho-1)}\right]^{1/\rho}} \tilde{S}. \tag{6.4.31}$$

Consequently, the amount of money spent on space by the representative consumer can be written as

$$\sum_{i=1}^{M}(p_i/a_i)(a_i S_i) = \left[\sum_{i=1}^{M}(p_i/a_i)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho} \tilde{S} = G\tilde{S}. \tag{6.4.32}$$

With

$$G = \left[\sum_j \left(\frac{a_j}{p_j}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}}, \tag{6.4.33}$$

as the *price index*, the budget constraint can alternatively be written as

$$Y = Z + G\tilde{S}. \tag{6.4.34}$$

From here on—in the second step—maximisation of the utility function (6.4.23) can be performed with (6.4.34) as the budget constraint. This is a standard maximisation problem and yields

$$V(Y,G) = \beta^{\beta}(1-\beta)^{(1-\beta)} YG^{-\beta}, \tag{6.4.35}$$

for the indirect utility function.

This function can be interpreted as the *social welfare function* for the population. Feenstra (1995, p. 636) points out that a demand function as in (6.4.38) can be derived from a representative consumer's indirect utility function, but states that this function is a *monotonic transformation* of "*... the sum over individuals of the expected value of maximised utility...*". It means that the aggregate welfare measure needs to be transformed before it can be interpreted as a utilitarian social welfare function. Here it is shown that if the indirect utility for the individual consumer follows a Cobb-Douglas specification in which the error term is added after a logarithmic transformation (see also Timmins, 2003), the representative consumer's indirect utility function is simply the sum of the individual utility functions. It follows that if the monotonic transformation is applied at the individual level, the indirect utility function of the representative consumer can be interpreted directly as the utilitarian social welfare function, without further transformation.

Comparing (6.4.33) with (6.4.21), it can be concluded that $\rho = \beta/(\beta+\mu)$. The evolutionary selection by a population of N agents of the equilibrium solution can therefore be considered equivalent to the solution to the problem for the representative consumer. In this case land is considered a differentiated good, while the representative consumer has preference for variety, expressed in a constant elasticity of substitution that is equal to $\sigma = (\beta+\mu)/\mu$.

For a given income level Y, the level of social welfare depends only on the *quality-adjusted price index* for space that is a function of both the price and the quality vector: $G = f(\mathbf{p},\mathbf{q})$. In principle, it can be considered a *green price index* that captures the combined impact of market prices and environmental quality in a single indicator (see also Banzhaf, 2005).

The total quality-adjusted amount of land demanded is given by

$$\tilde{S} = \frac{\beta Y}{G}. \tag{6.4.36}$$

Using (6.4.31), the quality-adjusted amount of land demanded at location j is equal to

$$\begin{aligned}\tilde{S}_j = a_j S_j &= \frac{\left(p_j/a_j\right)^{1/(\rho-1)}}{G^{1/(\rho-1)}}\tilde{S} \\ &= \frac{\left(p_j/a_j\right)^{1/(\rho-1)}}{\sum_{j=1}^{M}\left(p_j/a_j\right)^{\rho/(\rho-1)}}\beta Y.\end{aligned} \tag{6.4.37}$$

For the amount of land demanded (without quality adjustment), it follows that

$$\begin{aligned}S_j &= \frac{\left(p_j/a_j\right)^{\rho/(\rho-1)}}{\sum_{k=1}^{M}\left(p_k/a_k\right)^{\rho/(\rho-1)}}\frac{\beta N y}{p_j} \\ &= \frac{\left(p_j\Big/q_j^{\gamma/\beta}\right)^{-\beta/\mu}}{\sum_{k=1}^{M}\left(p_k\Big/q_k^{\gamma/\beta}\right)^{-\beta/\mu}}\frac{\beta N y}{p_j} \\ &= x_j\frac{\beta N y}{p_j} \\ &= N x_j\frac{\beta y}{p_j} \\ &= n_j s_j = A.\end{aligned} \tag{6.4.38}$$

Which is consistent again with the previous subsections.

6.4.4 Nested logit

A final observation concerns the interpretation of the set of location decisions taken by the representative consumer in terms of a logit model, in the same way as was derived in chapter 4. It will be shown that the corresponding model is a *nested* logit model (Anderson and de Palma, 1992; Train, 2003). The choice by a single agent can be represented as shown in figure 6.5. The Cobb-Douglas utility function (6.4.23) can be interpreted as a special case of a general CES utility function,

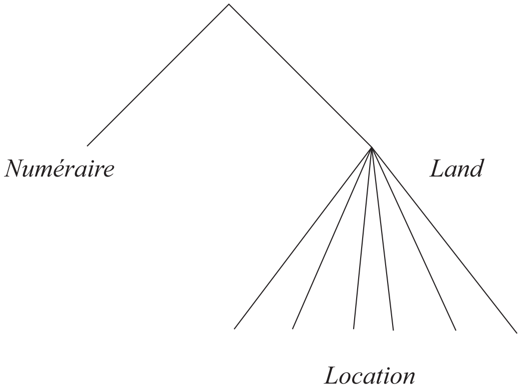

Figure 6.5: *Nested logit location choice for representative consumer.*

$$U(S,Z) = \left[\beta \tilde{S}^{\rho_1} + (1-\beta) Z^{\rho_1}\right]^{1/\rho_1}, \tag{6.4.39}$$

with $\rho_1 \to 0$.

The fraction of the total income the representative consumer spends on space at location j is equal to the fraction of income spent on space times the *fraction of the fraction* of the income spent on space. The amount of the total income spent on space in spatial equilibrium should be equal to $\Pr_{S_j} = \Pr_S \Pr_{j|S}$ and the demand at location j is equal to

$$S_j = \frac{\Pr_{S_j} Y}{p_j}. \tag{6.4.40}$$

In terms of probabilities, $\Pr_{j|S}$ can be read as the conditional probability; conditional on the probability that the entire income is spent on space. In the Cobb-Douglas case this product of probabilities was equal to βx_j. For the general specification (see Anderson and de Palma, 1992), the fraction of the income spent on space reads

$$\mathrm{Pr}_S = \frac{\exp\left(\frac{\ln\beta^{1+\mu_2}+\ln I_S}{\mu_2}\right)}{\exp\left(\frac{\ln\beta^{1+\mu_2}+\ln I_S}{\mu_2}\right)+\exp\left(\frac{\ln(1-\beta)^{1+\mu_2}+\ln I_Z}{\mu_2}\right)}. \tag{6.4.41}$$

The fraction of the fraction spent on land is the same as before:

$$\mathrm{Pr}_{j|S} = \frac{\exp\left(\frac{\ln a_j-\ln p_j}{\mu_2}\right)}{\sum_{k=1}^{M}\exp\left(\frac{\ln a_k-\ln p_k}{\mu_2}\right)}. \tag{6.4.42}$$

In (6.4.41), the following definition was used

$$\begin{aligned}\ln I_S &\equiv \mu_2 \ln\sum_{j=1}^{M}\exp\left(\frac{\ln a_j-\ln p_j}{\mu_2}\right)\\ &= \mu_2\ln\sum_{j=1}^{M}\left(a_j/p_j\right)^{1/\mu_2}\\ &= -\ln\left[\sum_{j=1}^{M}\left(p_j/a_j\right)^{\rho_2/(\rho_2-1)}\right]^{(\rho_2-1)/\rho_2}\\ &= -\ln G. \end{aligned}\tag{6.4.43}$$

Here, use has been made of the fact that $\mu_2 = (1-\rho_2)/\rho_2$. A comparison with (6.4.21) reveals that

$$\ln I_S = -\ln G. \tag{6.4.44}$$

Together with $\ln I_Z = \ln 1 = 0$, which corresponds to the role of the numéraire, the nested logit formulation highlights the role of the price index in a two-stage optimisation procedure. The first stage determines the part of the income spent on quality-adjusted land; that part is distributed over the locations in the second stage. This interpretation can also be applied in case of a population, if in the first stage the probability is set equal to the fraction every individual spends on land, for example as a time average as in chapter 4. The second stage could in that case be identified again with the distribution of individual agents over the locations, as in this chapter.

6.5 Sorting

The model developed in this chapter can relatively easily be adapted to enforce segregation of income classes using only endogenous prices and no externalities. This segregation can be interpreted as an extension of the concept of self-organisation to population of heterogeneous agents. Segregation by income is important in the original sorting models because this property corresponds to *stratification* in econometric estimation. Stratification implies that individual characteristics—in the model presented here restricted to income—can be enforced to correlate with location characteristics in the final equilibrium, in the presence of endogenous prices. It allows the researcher to differentiate the benefits from changes in the level of amenities according to, for example, different income groups. This distributional aspect of spatial welfare will be discussed in chapter 8. Endogenous sorting can be illustrated using a variant of the evolutionary Alonso variant of section 6.4. It has to be assumed that the preference structure depends on the individual characteristics. In the simplified model, in addition to the idiosyncratic component ε_{ij}, the only individual characteristic is the income. A specification of this dependency that conforms to the possibility of econometric estimation, is

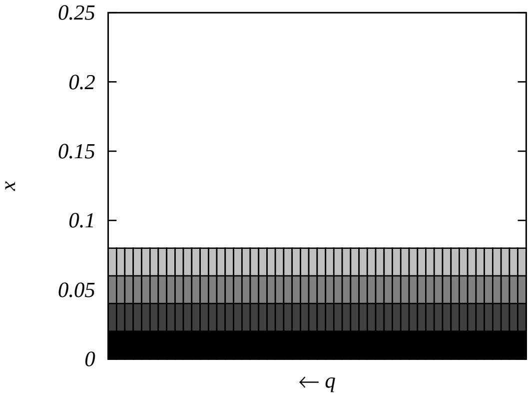

Figure 6.6: *Initial distribution of sorting model with 4 sub-populations.*

$$\begin{aligned}\ln v_{ij} &= \ln y_i - \beta_1 \ln p_j + \gamma_1 \ln q_j \\ &\quad - \beta_2 \ln y_i \ln p_j + \gamma_2 \ln y_i \ln q_j \,.\end{aligned} \tag{6.5.1}$$

It implies the following definitions of individual-level coefficients:

$$\beta_i = \beta_1 + \beta_2 \ln y_i, \tag{6.5.2a}$$

$$\gamma_i = \gamma_1 + \gamma_2 \ln y_i. \tag{6.5.2b}$$

Simulation results—following numerical integration—are presented in figures 6.6-6.8. Here the population is divided into four groups of equal size. Disequilibrium prices are determined according to (6.3.47), with the total amount of money offered for a location defined as the sum of the offers from the fractions of all four groups. With k as the index for the group, the price at location j can be written as

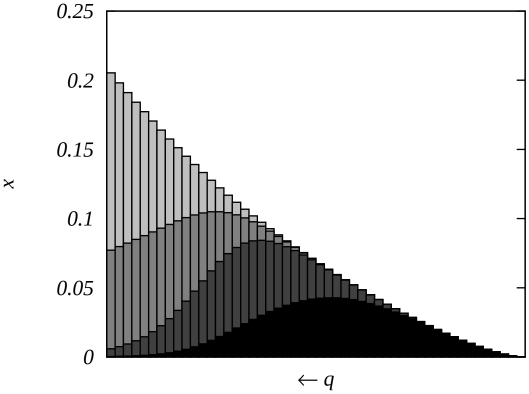

Figure 6.7: *Equilibrium type I for figure 6.6 ($\beta_2 < 0$).*

$$p_j = \frac{1}{2A} \sum\nolimits_{k=1}^{4} x_{j,k} \beta_k N_k y_k. \tag{6.5.3}$$

Starting with a uniform distribution of all groups shown in figure 6.6, two possible equilibrium configurations can arise, depending on the *sign* of β_2 in (6.5.2a). The value of γ_2 in (6.5.2b) is kept at zero, to show the effect of sorting by income only. In figure 6.7, β_2 is negative. It means that the group with a higher income spends a lower percentage of their income on housing than the lower income groups. The lower income groups consider space more important than the higher income groups. In this example, the actual amount of money spent on housing is nevertheless still higher for higher income groups than for the lower ones. The result is a concentration of the higher income groups closer to the CBD. Figure 6.8 shows the opposite. If higher income groups find space more important, they will position themselves away from the CBD. In this way, the models show an example of endogenous segregation according to individual characteristics.

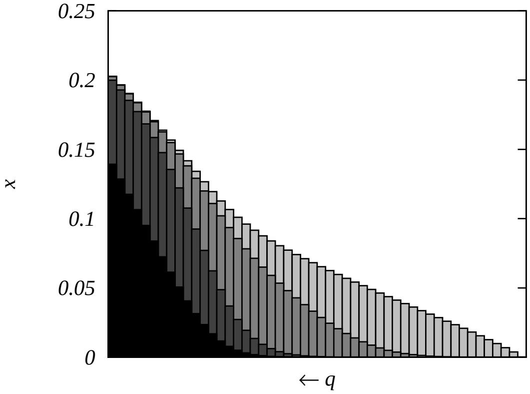

Figure 6.8: *Equilibrium type II for figure 6.6 ($\beta_2 > 0$).*

6.6 Conclusions

The main result of this chapter is the derivation of a consistent relation between an evolutionary interpretation of the Alonso model from urban economics and a representative consumer. First, the dependence of the level of well-being on the distance to the CBD through transport costs in the original Alonso model was replaced by a more general dependence through an exogenous amenity. This adjustment was followed by the introduction of discrete, instead of continuous, locations. The discrete case was finally reinterpreted as a population game based on the material concerning evolutionary game theory collected in chapter 3.

Although the discrete and continuous cases in this chapter were restricted to a Cobb-Douglas utility function, it was shown in section 6.4, that the discrete case can be generalised to a CES utility function for a differentiated good. This formulation has the advantage that it allows for an interpretation of a stochastic variant of a population game, while the social welfare function can also be interpreted as the indirect utility function of a representative consumer. The first interpretation can serve as the basis for an agent-based model, since the evolutionary dynamics already defined the behavioural rules at the level of individuals. The latter facilitates the welfare analysis for simplified cases.

Finally, the evolutionary assessment of a traditional agglomeration model as the result of self-organising individuals was extended to a model with four sub-populations. It was shown that with a preference structure that is dependent on the individual—or in this case group—characteristics, it is possible to enforce segregation of the groups by means of endogenous prices only. This result corresponds to the stratification of income groups in econometric sorting models. Endogenous sorting can be used in a differentiation of benefits from changes in local amenity levels according to income, as will be shown in chapter 8.

6.7 Appendix

This appendix derives the analytical solution for the population frequency distribution of (6.4.14). The starting point is (6.4.11), repeated here as

$$x_j = \frac{\left(q_j^\gamma \big/ p_j^\beta\right)^{1/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ p_k^\beta\right)^{1/\mu}}. \tag{6.7.1}$$

Substitution of (6.3.47) in (6.7.1) results in

$$x_j = \frac{\left(q_j^\gamma \big/ x_j^\beta\right)^{1/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}}. \tag{6.7.2}$$

Reordering yields

$$x_j^{1+\beta/\mu} = \frac{q_j^{\gamma/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}}, \tag{6.7.3}$$

or

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\left[\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}\right]^{\mu/(\beta+\mu)}}. \tag{6.7.4}$$

Finally, using the identity

$$\sum_{j=1}^M x_j = 1, \tag{6.7.5}$$

allows the denominator of (6.7.4) to be written as

$$\left[\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}\right]^{\mu/(\beta+\mu)} = \sum_{k=1}^M q_k^{\gamma/(\beta+\mu)}. \tag{6.7.6}$$

Substituting back in (6.7.4) solves for the population frequency:

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}. \tag{6.7.7}$$

Chapter 7

Welfare analysis in spatial equilibrium

7.1 Introduction

The population model developed in chapter 6 defines a social welfare function for a population of agents in a spatial equilibrium. The welfare level in spatial equilibrium plays a role in both urban economics and hedonic pricing. This chapter employs the model from chapter 6 in several types of welfare analyses based on the interpretation of the model as a stylised, theoretical version of a *locational sorting model*. As discussed in chapters 1 and 5, locational sorting models are recently introduced location choice models that help extend hedonic pricing methods for the valuation of the benefits from non-marginal changes in local environmental quality. Palmquist (2004, p. 59) notes that:

> *'The theoretical hedonic model describes an equilibrium, but there has been little formal work on modelling how that equilibrium would change if there were changes in exogenous factors.'*

Locational sorting models primarily focus on the welfare effects of the capitalisation of the value of changes in exogenous amenities in the market price for land. This chapter contributes to the understanding of the price equilibrium in hedonic pricing in terms of a spatial equilibrium.

The price for land in locational sorting models is determined in a market equilibrium that resembles the Alonso model in urban economics, though it is usually with a fixed total supply of residential space. By adding the notion of total amount of consumed land from urban economics, both quality and quantity aspects of land use are combined in a single consistent welfare measure. In that sense, this allows for a welfare analysis that simultaneously addresses two of the three welfare issues discussed in chapter 1, related to land use and concerning land policy:

1. the optimal distribution of residential space,
2. the distribution of local public goods, or amenities.

Furthermore, adopting the interpretation introduced in section 6.4, a simplified sorting model can be considered a model of a representative consumer. In that model, the notion of spatial welfare might be captured in a single indicator: the price index for quality-adjusted land prices. This index will be shown to be useful in the interpretation of a so-called general equilibrium willingness to pay for either change in local amenity levels or the number of locations developed for residential use. It will be demonstrated that the dependency of the welfare function on the number of developed locations can in theory be operationalised in the assessment of *open space* as a pure public good. The welfare contribution of open space was identified in chapter 1 as the third issue in land policy.

This chapter is organised as follows. Section 7.2 discusses the relation between locational sorting models and hedonic pricing. Section 7.3 discusses the efficient allocation of land in the presence of simple agglomeration externalities that can be interpreted as network externalities, analogous to the discussion in chapter 4. How the model can be used in an assessment of a tendency to urban sprawl is illustrated in section 7.4. Section 7.5 discusses in detail the problem of dealing with the mutual exclusiveness of use and non-use of land in a consistent welfare measure. This concerns the effective contribution of open space to the level of social welfare in an agglomeration. Finally, section 7.6 concludes this chapter.

7.2 Hedonic pricing and locational sorting

This section discusses the original motivation for locational sorting models. Starting with the concept of hedonic pricing, it will be shown that a strict interpretation of the marginal change in the price as a marginal willingness to pay would imply that agents do not move after the distribution of local public goods is changed.

7.2.1 WTP for marginal changes

In a market equilibrium resulting from utility maximisation, Marshallian demand will equal Hicksian demand, since it is assumed that the equilibrium level of utility, u^*, is fixed and at the maximum level. If this level also corresponds to the utility level in spatial equilibrium, the demand can be written as

$$s^* \left(q_j, u^*\right) \equiv s^M \left[p \left(q_j, u^*\right), y\right] \equiv s^H \left[p \left(q_j, u^*\right), u^*\right]. \tag{7.2.1}$$

Resulting from the first order conditions, the bid rent in equilibrium is essentially equal to the *price*, or the marginal rate of substitution; therefore

$$p_j^* = -\frac{\partial u^* / \partial s}{\partial u^* / \partial z} = -\frac{\partial z_j^*}{\partial s}. \tag{7.2.2}$$

This observation leads to the following justification of using hedonic prices to derive a willingness to pay (WTP) for a *marginal* change in amenity level q_j, with the equilibrium amount of space, s_j^*, kept fixed. Starting with the assumption that a change in the amenity level affects both the demanded quantity of land, s, and—through the budget constraint—the demand for the numéraire, the marginal change in the price following a marginal change in the quality level can be decomposed as (Scotchmer, 1986 and Haab and McConnell, 2002, p. 250)

$$\frac{\partial p}{\partial q} = \frac{\partial p}{\partial s}\frac{\partial s}{\partial q} + \frac{\partial p}{\partial z}\frac{\partial z}{\partial q}. \tag{7.2.3}$$

With the price taken from the budget constraint, $p = (y - z)/s$, and keeping the amount of land fixed, the first part of the right-hand side is equal to zero and (7.2.3) reduces to

$$\frac{\partial p^*}{\partial q} = -\frac{1}{s^*}\frac{\partial z}{\partial q}. \tag{7.2.4}$$

In terms of small, but finite, changes, (7.2.4) can be interpreted as

$$s^* \Delta p^* + \Delta z = 0. \tag{7.2.5}$$

Using as a definition $WTP \equiv -\Delta z$, it follows from (7.2.5) that this WTP is sufficient for compensating the change in price, Δp^*, following the change in quality, according to

$$y - WTP = (z + \Delta z) + (p^* + \Delta p^*)\, s^* = z + p^* s^*. \tag{7.2.6}$$

As a result,

$$v\,(y - WTP, p^*; \hat{q}) = v\,(y, p^*; q) = u^*. \tag{7.2.7}$$

From (7.2.1) it follows that the WTP in (7.2.7) still conforms to a Hicksian consumer surplus. As noted in chapter 5, this measure is used extensively in environmental economics in various valuation methods for the valuation of changes in public goods. In this sense, the interpretation of hedonic pricing in spatial equilibrium conforms to the general approach adopted in valuation methodology.

The WTP for an amenity improvement $\Delta q = \hat{q} - q$ for tenants, is equal to the change in price, Δp^*, that can be derived from the hedonic regression on the quality, q, times the equilibrium quantity, s^*, consumed. For a marginal change, 7.2.7 is always valid. For a non-marginal change, however, it would imply that local equilibrium rent and equilibrium demand for space remains unchanged. In terms of a rental market for space, this assumption translates to the *condition* that the individuals in the population will not move to other locations in response to changes in q. Hence the interpretation follows that moving costs are sufficiently high to prevent the population from *resorting*. This has inspired the search for a WTP based on endogenous market equilibrium prices, $\mathbf{p}$, or *general equilibrium willingness to pay* (GE-WTP) (Smith et al., 2004). The following subsection is based on relating the utility level in spatial equilibrium, u^*, to a GE-WTP.

7.2.2 General equilibrium willingness to pay

Locational sorting models contribute to the valuation literature by defining a general equilibrium willingness to pay (GE-WTP) per individual by (Smith et al., 2004):

$$v\,(p_l^*, y - WTP_{GE}; \hat{q}_l) = v\left(p_j^*, y; q_j\right). \tag{7.2.8}$$

Here, p_l^* denotes the equilibrium price corresponding to a change from vector (of all locations) $\mathbf{q}$ to $\hat{\mathbf{q}}$, where the index l signals that the location choice for the individual might have changed in the new equilibrium. This GE-WTP is contrasted with the general definition of a Hicksian (partial equilibrium, or short-run) WTP for changes in the quality level of a public good only, keeping equilibrium prices, $\mathbf{p}^*$, fixed:

$$v\left(p_j^*, y - WTP_{PE}; \hat{q}_j\right) = v\left(p_j^*, y; q_j\right). \tag{7.2.9}$$

While (7.2.8) intuitively makes sense, the problem is to find the new location choices and the new equilibrium prices. The value of WTP_{GE} critically depends on the definition of the new market equilibrium. In short, a mechanism needs to be designed that derives consistent values for p_l^*. This mechanism is then applied to calculate the equilibrium values of a counter-factual equilibrium (that is, the equilibrium with hypothetical changes in the values of the quality levels). In the literature on locational sorting, the total supply of housing is often taken to be fixed, assuming that the population would resort over the existing stock of houses. The specification of both demand and supply introduces endogenous prices, and thereby the definition of the 'general equilibrium,' in the model.

The assumption of a fixed supply resembles the fixed supply per location, A, in the discrete location choice model of section 6.4, together with a fixed number of locations, M. If consumers are assumed identical, the level of utility in a new market equilibrium, after changing the state from $\mathbf{q}$ to $\hat{\mathbf{q}}$, is given by the following indirect utility:

$$v_j\left[p_j^*\left(\hat{\mathbf{q}}\right), y; \hat{\mathbf{q}}\right] = u_j^*. \tag{7.2.10}$$

Section 6.4 also showed that the fixed supply per location—interpreted as a *no-vacancy constraint*—introduced the Nash equilibrium, while the Nash equilibrium implies the same level of utility at every location for identical agents. Hence, the market equilibrium bears all features of the spatial equilibrium u^*. When the market equilibrium of a locational sorting model is identified with a spatial equilibrium, it can be conjectured that a general equilibrium willingness to pay is likely to value the difference in utility of two spatial equilibria. In general, it is to be expected that

$$\hat{u}^* \neq u^*. \tag{7.2.11}$$

In other words, the counter-factual equilibrium is probably characterised by a different level of equilibrium utility. Against this background, the GE-WTP is mainly restoring the old utility level:

$$v\left[p^*\left(\hat{\mathbf{q}}\right), y - WTP_{GE}; \hat{\mathbf{q}}\right] = u^*. \tag{7.2.12}$$

Both connotations of the market equilibrium shed different lights on a GE-WTP. The relation with a hedonic bid rent facilitates an interpretation in terms of adjusting, or compensating, a pure Hicksian willingness to pay for the capitalisation of quality changes in the rent. From the perspective of a spatial equilibrium, the GE-WTP would be a monetary measure for comparing the welfare level of two different simultaneous distributions of agents and amenities. Therefore, a GE-WTP could also be read directly as a monetary measure for a change in *spatial welfare*.

Since the equilibrium value for spatial welfare, u^*, was given an evolutionary connotation in chapter 6, this value captured in (6.4.35) as

$$V(Y,G) = Nu^* = \beta^\beta (1-\beta)^{(1-\beta)} YG^{-\beta}, \tag{7.2.13}$$

allows for the interpretation of the model of chapter 6 as a stylised locational sorting model, from which a theoretical value for a GE-WTP can be derived. Furthermore, since the quality-adjusted price index, G, is also a function of the number of developed locations, M, the concept of a GE-WTP can be extended also to include a comparison of the welfare levels between two different land use configurations. This welfare measure would correspond to the basis of an idealised cost-benefit analysis assessing different land use options, as discussed in chapter 1.

7.3 Efficient allocation of land

The relation between the population model and the model for the representative consumer demonstrated in chapter 6 offers an interesting way to show that the spatial equilibrium is optimal, in the neoclassical sense. At the individual level, the expected level of utility was shown to be the same for every agent. In that respect, the spatial equilibrium seems to be Pareto optimal, because no agent can choose a better location without making another agent worse-off. However, it is possible—at least in principle—that all agents could be made better off with a different set of market land prices, similar to the discussion on efficiency in section 4.6. This is not the case with an exogenous quality level at each location, but with *endogenous* quality levels—interpreted as *external effects*—, improvement of the overall level of well-being is theoretically possible.

Chapter 6 showed that the level of utility at market equilibrium prices and the utility level in spatial equilibrium are the same. Based on the analogy with initial endowments in an exchange economy, it was assumed that the total available amount of space per location was fixed. This observation can be used to assess the optimality of the residential equilibrium for a city with M locations. The starting point is the social welfare function

$$\max_{\tilde{S},Z} U\left(\tilde{S},Z\right) = \tilde{S}^\beta Z^{1-\beta} \quad s.t. \quad Y = Z + G\tilde{S}. \tag{7.3.1}$$

The price index, G, in the budget constraint will not yet be specified. The appendix to this chapter shows how the utility function in (7.3.1) can indeed be considered a utilitarian social welfare function in the traditional sense, $U = Nu$, if $\rho = 1$. Since ρ was identified with the error term added on purpose to support a direct behavioural interpretation of the indirect utility function, a different approach is needed for $\rho \neq 1$.

First of all, it is observed that because the total available amount of land is fixed, $S_j = A$, for all locations j (no vacancy), the amount of the first good, the combination of quantities and qualities of land, $\tilde{S}$, is essentially fixed as well:

$$\begin{aligned} \tilde{S} = \left[\sum_{j=1}^{M} \tilde{S}_j^{\rho}\right]^{1/\rho} &= \left[\sum_{j=1}^{M} \left(q_j^{\gamma/\beta} S_j\right)^{\rho}\right]^{1/\rho} \\ &= A\left[\sum_{j=1}^{M} \left(q_j^{\gamma/\beta}\right)^{\rho}\right]^{1/\rho}. \end{aligned} \tag{7.3.2}$$

The only degree of freedom in optimising the utility function therefore concerns the numéraire,

$$Z = Y - A\sum_{j=1}^{M} p_j. \tag{7.3.3}$$

At this stage, the assumed market equilibrium prices for the locations can be written as

$$p_j = x_j \frac{\beta Y}{A}, \tag{7.3.4}$$

without specifying the fraction, x_j, of the total income, $Y = Ny$, yet. Substitution of (7.3.3) in (7.3.4) yields

$$Z = (1 - \beta)\, Y. \tag{7.3.5}$$

The demand in (7.3.5) always corresponds to the solution of (7.3.1), irrespective of the value of the price index, G, for quality-adjusted land and thereby independently of the fractions, x_j, in (7.3.4). This is in line with the Cobb-Douglas specification of the upper-level decision, concerning the division of the income between land and all other goods. And since the value of the entire vector of fractions, $\mathbf{x}$, was determined as a Nash equilibrium of a population game, an analysis similar to the one from section 4.6 can be applied here at the level of the individual agent.

In chapter 6, the evolutionary selection mechanism that resulted in the Nash equilibrium,

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}, \tag{7.3.6}$$

was based on indirect utility function

$$\ln v_j + \mu\varepsilon_j = \ln y - \beta \ln p_j + \gamma \ln q_j + \mu\varepsilon_j. \tag{7.3.7}$$

This indirect utility function can be interpreted as the logarithm of the solution to the maximisation of

$$\tilde{u}\left(s_j, z; q_j\right) = s_j^{\beta} z^{1-\beta} q_j^{\gamma} e^{\mu\varepsilon_j}, \tag{7.3.8}$$

with respect to land, s_j, and the numéraire, z, subject to the usual budget constraint, $y = z + p_j s_j$ and neglecting the constant term $\beta \ln \beta + (1-\beta) \ln (1-\beta)$ in (7.3.7).

Since both the quality, q_j, and the 'circumstances', ε, are exogenous, by virtue of the remaining Cobb-Douglas utility function, agents can maximise their utility for given prices $\mathbf{p}$. Because the Nash equilibrium (7.3.6) is *unique*, and optimal from an evolutionary perspective, the price vector representing the market equilibrium, $\mathbf{p}^*$, supports an efficient allocation of land. The distribution of agents represented by 7.3.6 guarantees that the expected level of utility is the same at every location[1].

7.3.1 Efficiency in the presence of agglomeration externalities

In the presence of an agglomeration externality, that is formalised as a *network externality* similar to the one introduced in 4.6, a difference will appear between the market equilibrium and the social optimum. In addition to the exogenous quality, q_j, an *endogenous location quality*, x_j, will be added. This endogenous amenity is equal to the local population density and it represents a simple agglomeration effect, as it attracts agents to locations where other agents are present. Because it is only a local effect, it is insufficient to enforce an endogenous CBD, but it does allow for a simple assessment of the internalisation of the value of an endogenous amenity in the value of land.

If the local population density is considered a real externality, the total amenity level—the combination of q_j and x_j—will remain exogenous, not affecting her optimisation problem. The logarithm of the corresponding indirect utility function is

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[1] In the original Alonso model, equal levels of utility need to be treated as an additional condition. According to Fujita and Thisse (2002, p. 84), a *Rawlsian welfare function* is therefore needed in the assessment of the optimality of the allocation of land.

given by

$$\ln v_j + \mu\varepsilon_j = \ln y - \beta \ln p_j + \gamma \ln q_j + \delta \ln x_j + \mu\varepsilon_j. \tag{7.3.9}$$

The endogenous amenity now appears as a direct dependency on the strategic choices of other agents. This formalisation conforms to the specification of a multinomial choice model with social interactions (Brock and Durlauf, 2003). It can be inserted in a numerical solution procedure, for example the set of evolutionary differential equations for the CES dynamics in section 6.4.

The social planner, however, could in principle optimise the individual amount of land, also taking into account this externality. With, $x_j = p_j s_j / y$, the direct utility function for the individual, as faced by the social planner can be written as

$$\tilde{u}\left(s_j, z; q_j\right) = s_j^{\beta+\delta} z^{1-\beta} q_j^{\gamma} \left(p_j / y\right)^{\delta} e^{\mu\varepsilon_j}. \tag{7.3.10}$$

Using the same approach as in section 4.6, the optimal demand is given by

$$s_j = \left(\frac{\beta+\delta}{1+\delta}\right) \frac{y}{p_j}. \tag{7.3.11}$$

Substitution of the demand functions in (7.3.10), results in an indirect utility function that is the same as (7.3.8). As noted in section 4.6, demand in (7.3.11) is higher than in case external effects are not internalised. This observation can be used in the definition of a price that yields the same demand as in (7.3.11):

$$\tilde{p}_j = \beta \left(\frac{1+\delta}{\beta+\delta}\right) p_j. \tag{7.3.12}$$

It follows that the optimal price—the price of (7.3.11) in market equilibrium—, denoted $\tilde{p}_j^*$, will be lower than the market price if the local density is kept fixed.

For a fixed total population size, N, and allowing for the number of locations, M, to be endogenous—as in section 6.4 by using the condition reflecting opportunity costs, $p_j \geqslant p_A$,—, lower prices imply a lower M. This is because the optimal price will drop below the opportunity costs from alternative use at the boundary of the agglomeration. This is illustrated in figures 7.1 and 7.2. An optimal allocation of land in the presence of agglomeration externalities would therefore imply that agglomerations should be smaller than a market allocation. This corresponds to the general notion that if positive externalities are internalised in the price of land an efficient allocation of land will yield a smaller agglomeration than in market equilibrium (Fujita and Thisse, 2002, p. 179-182). This section shows that this result can be applied to models with local social interactions, interpreted in terms of land use models in traditional urban economics. It can be concluded that a distinction arises

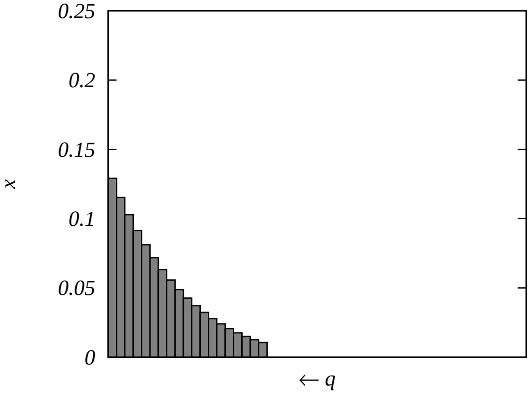

Figure 7.1: *Alonso variant with network externalities not internalised.*

between the socially optimal and the market equilibrium for models that allow for agglomeration externalities—or *social interactions*. In the presence of externalities, the market allocation generally results in an oversupply of land.

However, the strict internalisation of endogenous amenities in terms of external effects as in the neoclassical framework, would fail to do justice to the interpretation of the quality aspect. Capitalisation of the value of an exogenous quality level is in accordance with an optimal allocation of land. The question arises of what determines the 'inefficiency' of the allocation, if the value of an endogenous quality level is capitalised analogously. Put differently, a neoclassical perspective on the allocation of land would only 'accept' exogenous amenities. Many types of amenities—other than the network externalities presented here—are likely to depend on the location choices of other agents, especially in the formation of land use patterns. The contribution of this interdependency to the level of spatial welfare can in theory be captured by the measures used in this chapter, as long as prices in the market equilibrium are accepted as they are. This applies, however, only for the locations for which residential use is considered optimal, as will be shown in section 7.5 and chapter 8 in the discussion on open space.

7.4 Prioritising

Another, relatively simple, welfare analysis can be based on the social welfare function, (6.4.35), of section 6.4. The value of 'marginal resorting' can be used to es-

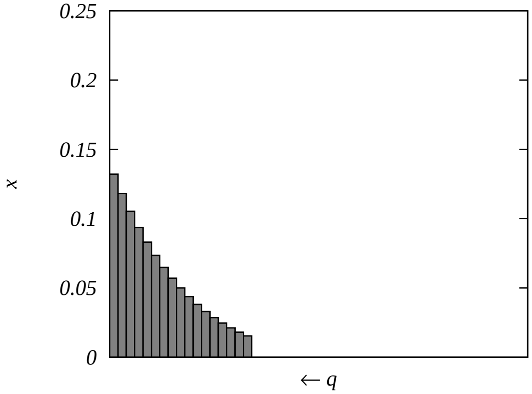

Figure 7.2: *Alonso variant with network externalities internalised.*

tablish at which location the quality level should be improved to yield the highest general equilibrium benefit for the entire population.

Differentiation of the social welfare function (6.4.35) with respect to q_j results in

$$\frac{\partial V}{\partial q_j} = \gamma A^{\beta} \left[(1-\beta)Y\right]^{1-\beta} \left(\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta+\mu}}\right)^{\beta+\mu-1} q_j^{\frac{\gamma}{\beta+\mu}-1}. \tag{7.4.1}$$

It follows from (7.4.1) that the location for which a marginal change in the amenity level results in the highest marginal change of the social welfare function depends on

$$\frac{\gamma}{\beta+\mu} \lesseqgtr 1. \tag{7.4.2}$$

The only term in (7.4.1) different for every location is $q_j^{\frac{\gamma}{\beta+\mu}-1}$. If $\frac{\gamma}{\beta+\mu} > 1$, a marginal change in quality for the location with the *highest* amenity level will yield the largest effect on social welfare. If $\frac{\gamma}{\beta+\mu} < 1$, the value of $q_j^{\frac{\gamma}{\beta+\mu}-1}$ will be determined by $1/q_j$. This implies that in this case a change in quality at the location with the *lowest* level will have the largest impact on social welfare.

This result can be explained as follows. Assuming that the idiosyncratic element in the preference structure does play a role, $\mu \downarrow 0$, prioritising according to (7.4.2) basically depends on $\gamma \lesseqgtr \beta$, or the ‘weight’ of the *price*, β, relative to the ‘weight’ of the *amenity*, γ, in the preference structure (7.3.9) of all households. Since the level of utility is the same at every location, the welfare impact can also be analysed at locations where there is no change in amenity level, but only a change in price or—according to (7.3.4)—equivalently in *population density*. A higher welfare level can only be achieved at these locations if a part of the population leaves the location. Because the size of the total population, as well as the number of locations, is fixed, agents can only migrate to the improved location.

Focusing again on the district where there is an increase in amenity level, the net increase in welfare for this district reflects a trade-off between quality improvement and the 'dis-utility' of the higher population density following the migration of people attracted to the improved location. The trade-off is apparently optimal at the location with the lowest amenity level if $\gamma < \beta$ and at the location with the highest amenity level if $\gamma > \beta$. Because of (7.3.6), in equilibrium: $x_j \sim q_j$. Therefore the location with the lowest amenity level has the lowest population density. In conjunction with the corresponding direct utility function for the individual agent, $u_j = s_j^{\beta} z^{1-\beta} q_j^{\gamma}$, it can be concluded that if space is preferred over quality ($\gamma < \beta$), the negative impact of an increased population density will be minimal at the location where the population density is lowest. By the same argument, if quality is preferred over space ($\gamma > \beta$), the negative impact of an increased population density will be minimal at the location where the existing quality level is highest.

Because of the large number of simplifications in the model, one has to be careful when drawing conclusions for policy implications. Nonetheless, of the two possibilities in (7.4.2), the preference of space over quality seems to reflect the tendency of urban sprawl in cities, or at least the tendency of people to move to suburbs or nearby villages. A municipal policy offering more facilities in the city centre—as the location with the highest population density—as a counter measure, is likely to have a limited effect in this case.

7.5 Open space as amenity

In the framework used thus far, land as a market good is combined with a local pure public good that can be interpreted as the local quality level. With respect to the discussion on elements needed for a framework for addressing a socially optimal allocation of land in chapter 1, the focus until now has been only on land used for a commercial purpose. This section adapts the framework for the integration with *open space*. If open space is considered an amenity, its social value can be derived using valuation methods developed in environmental economics, for example contingent valuation or hedonic pricing. In that case, open space is a public good under-supplied by markets and the state would be called to intervene. The protection of open space could in this perspective be interpreted as the direct supply by the government of a good that contributes to the well-being of all consumers, but which cannot be allocated efficiently by markets.

In terms of land use, however, open space is an alternative use to other land use types such as residential space and agriculture. Even if open space is interpreted analogously to a natural capital stock as in resource economics, this ‘non-use’ type competes with other land use types. If residential space is a market good bought by consumers, the production of it can be viewed as a land conversion process. In that case, the land owner—or landlord—obtains a position of a producer in the modelling framework. The conversion process is captured by a production function that uses land as natural capital as the production factor (input) for the production of residential space (output). This perspective not only highlights the mutual exclusiveness of use and non-use due to the same production factor, it also allows for a consistent interpretation of rent, both in terms of the rental price for residential space and the income from capital for the land owner. With open space as a natural capital stock, an interpretation as pure public good is possible still. It could for instance be justified to treat the volume of the capital stock as a factor that directly influences the level of well-being (utility) of individuals positively. A similar concept is commonly applied to a pollution stock in some resource economics frameworks, where the stock has a negative impact (Dasgupta and Mäler, 2000). Treatment only as a capital stock, however, does not address the exclusiveness of non-use vs. use.

To focus the discussion, this section distinguishes between the local amenity value capitalised in the price of land and open space as a pure public good with status as natural capital stock. The difference is that at the location of the former the land use type is still residential, while at the latter it essentially concerns non-use. This approach marks an important difference with a related assessment by Wu and Plantinga (2003). There, open space is treated as a local amenity of negligible size. Closer to the analysis in this paper is the work of Strong and Walsh (2005), where open space has a quantity measure that competes with residential space. The difference is that Strong and Walsh (2005) assume that welfare impacts result from spillover effects instead of a resource stock. The main issue is under what conditions open space can be protected. Often the loss of open space is considered a public ‘bad’ (Dasgupta and Mäler, 2000, p.73). If the well-being of a society is affected by both the presence of open space and the consumption of space for residential purposes, the question arises as to how the two can be balanced. Given the fact that location choices reflect a demand, a paradox seems to occur as in theory people would not choose to live at the cost of open space if they knew that its loss would negatively affect their utility level. This perspective seems to be supported by Polomé et al. (2005). They show that in a contingent valuation study conducted in the Netherlands, the respondents appear concerned about the space that protected areas require at the expense of other land use types, because the WTP for an additional protected area decreases if the total number of protected areas exceeds three.

One can argue that the destruction of open space has a prisoner's dilemma or free rider quality, where some choose to enjoy open space by being the first to actually live inside of it, causing a chain reaction. This section, however, opts for a different interpretation, as the paradox seems persistent even when the conservation of open space is be enforced by zoning.

7.5.1 Problem statement

If it is assumed that the price index, G^*, reflects the equilibrium price-quality levels of a given M residential locations, an expansion of residential land use to $M+1$ locations, using the social welfare function (7.2.13), the welfare effect would be captured in

$$V\left(Y,G^*\right)=V\left(Y-N\cdot WTP_{GE},\hat{G}^*\right). \tag{7.5.1}$$

If the expansion to $M+1$ residential locations takes place at the expense of a pure public good, both effects need to be combined in one WTP. This is in essence the consequence of the mutual exclusiveness of land use types. Given an area that could be perceived as pure public good, the welfare measure valuing 'non-use' expresses *existence value*. No existence value can be measured if the same area is used for residential purposes. Instead, the welfare contribution of an additional amount of land—as an *consumption good* together with the local amenity—will need to be valued. The following approximation is proposed:

$$V\left(Y+N\cdot WTA_{GE},G^*\right)=V\left(Y,\hat{G}^*\right). \tag{7.5.2}$$

Expression (7.5.2) maintains that a *general equilibrium willingness to accept* (GE-WTA) is assumed to capture the required monetary-equivalent compensation that results in the same level of social well-being, otherwise achieved by expanding the residential area. In other words, this GE-WTA reflects the minimum value of nature at the location (with the index $M+1$) to be developed. The GE-WTA therefore indirectly defines the value of open space as a pure public good.

This value is only suitable in the context of a cost-benefit analysis (CBA) if it can play a role in the decision of whether or not to develop. Prevention from development—or *protection* of nature at that location—would be feasible in an urban land use context, if the GE-WTA can be related to *rent* from 'non-use', or

$$p_{l,p}A_l\equiv N\cdot WTA_{GE}, \tag{7.5.3}$$

where the subscript l, p of the land price p refers to protection at location l. The location l is assumed to be of size A. Finally, the condition to be tested is

$$p_{l,p} \geq p_{l,r}. \tag{7.5.4}$$

The CBA thereby addresses the following question: Does the rent identified with the GE-WTA in (7.5.3) exceed the rent that the land owner could receive as a market rent from residential use by tenants? Condition (7.5.4) is inspired by urban economics literature where a similar expression determines the city border as opportunity costs for the land owner. If in this model it is assumed that society owns all the undeveloped land—stressing its 'public' character—, adopting (7.5.3), $p_{l,p}$ in (7.5.4) the value of nature might be thought of as *social opportunity costs*. Condition (7.5.4) maintains that while society would be as well off protecting the area as developing it, market-like conditions would enforce protection.

7.5.2 Solution

Welfare considerations can be based on the corresponding indirect utility function:

$$V = YG^{-\beta}. \tag{7.5.5}$$

Expression (7.5.5) can be used instead of (7.2.13), because only differences in welfare levels are to be accounted for and the constant terms can be neglected. Going back to (7.5.2), the aggregate WTA can be written, using (7.5.5) as

$$N \cdot WTA_{GE} = Y \left[\left(\frac{\hat{G}^*}{G^*} \right)^{-\beta} - 1 \right]. \tag{7.5.6}$$

Using (7.5.6), condition (7.5.4) implies

$$p_{M+1,r} = \beta Y x_{M+1} \leq Y \left(\frac{\hat{G}^{*-\beta}}{G^{*-\beta}} - 1 \right) = p_{M+1,p}. \tag{7.5.7}$$

In terms of the population frequency at the potentially developed location $M+1$, the condition can be rewritten (see the appendix) as

$$\beta x_{M+1} + 1 \leq \left[\frac{1}{1 - x_{M+1}} \right]^{\beta+\mu}. \tag{7.5.8}$$

Defining f according to

$$f \equiv (\beta x_{M+1} + 1) \left(1 - x_{M+1}\right)^{\beta+\mu}, \tag{7.5.9}$$

the condition $f \leq 1$ can be verified graphically in figure 7.6 for several ranges for x_{M+1} and β. Fulfilment of the condition implies that the virtual rent from protecting the location from development is indeed higher than residential rent. Protection therefore can—in theory—be enforced if the price for land at any location l is defined as the maximum of residential and 'protection' rent:

$$\tilde{p}_l \equiv \max\{p_{l,p}, p_{l,r}\}. \tag{7.5.10}$$

This rent should be taken into account by the individual, altering (7.3.8):

$$\ln v_j = \ln y - \beta \ln \left[\max\{p_{j,p}, p_{j,r}\}\right] + \gamma \ln q_j + \varepsilon_{ij}, \tag{7.5.11}$$

where

$$p_{l,p} \equiv \frac{N \cdot WTA_{GE}}{A}. \tag{7.5.12}$$

With (7.5.11) in the individual choice problem, the location valued in the WTP will be protected. The equilibrium population distribution will in that case resemble figure 7.4.

This result can be interpreted as follows. Instead of making explicit the contribution of open space to social and individual well-being, an individual additional virtual income is defined that originates as a rent from open space as a natural capital stock. Given this additional income, individuals are indifferent between extending the total residential area and protecting the open space. Direct comparison of the virtual rent with real rent from residential use, shows that the virtual rent will always be higher. If the individual agent considers the virtual rent as opportunity costs, she would opt for protection.

The model developed here strictly assumes that the owners of the residential areas are absentee landlords, following the urban economics tradition. Due to the Cobb-Douglas specification of the aggregate direct utility, however, the total amount spent on land will always equal βY, independent of the number of developed locations. This means that the result will remain valid if public ownership of the residential area is assumed, where all individuals earn an equal share of the aggregate rent (cf. Fujita, 1989, ch.3). Such an interpretation would fully comply with the interpretation of the virtual rent as opportunity costs. Using the welfare measure suggested in this section, the monetary value of one additional developed location can be derived. If this value is interpreted as the social opportunity costs, reflecting a virtual income from rent on open space as a natural resource, these opportunity costs are shown to exceed the market value of the same location based on income residential rent. This might resolve the apparent paradox that arises when it is suggested that consumers can in principle control the amount of open space by generating the

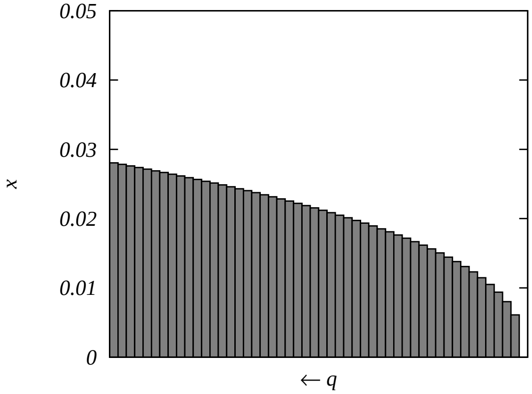

Figure 7.3: *Equilibrium distribution of agents without open space.*

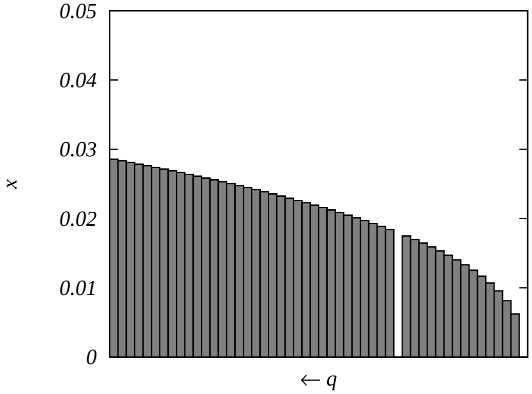

Figure 7.4: *Equilibrium distribution of agents with open space.*

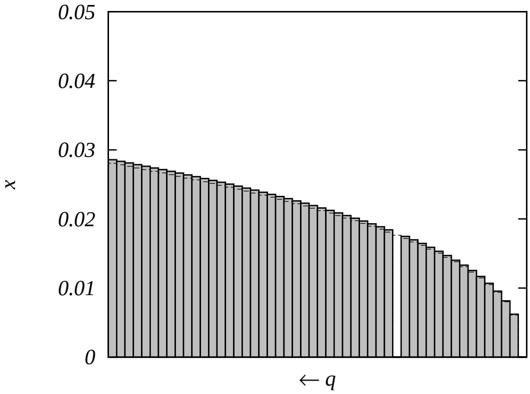

Figure 7.5: *Figures 7.3 and 7.4 compared.*

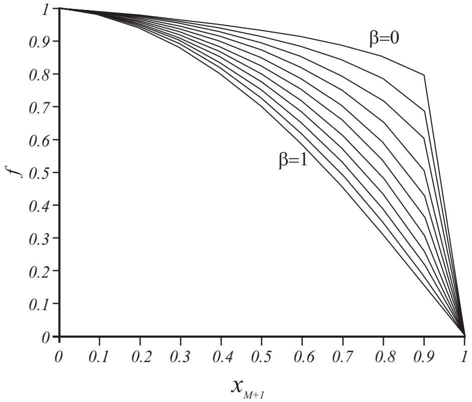

Figure 7.6: *Plot of* (7.5.9) *for different values of* β*.*

appropriate demand in their location choices. If individual agents take these opportunity costs into account in their location decisions, open space would be protected. This result suggests an alternative approach to existing valuation methods for estimating the benefits of open space, while taking into account the mutual exclusiveness of land use versus non-use.

7.6 Conclusions

This chapter used the model developed in chapter 6 as the basis for different types of welfare analysis. Because the equilibrium solution for this model can be identified with the model of a representative consumer, it facilitates the interpretation as a theoretical version of a locational sorting model. Locational sorting models are used in property valuation as an extension of hedonic pricing. Section 7.2 demonstrates how the level of welfare in locational sorting models can be interpreted in terms of land use models in urban economics. Generally only the willingness to pay (WTP) for marginal changes in the local quality level can be derived in hedonic pricing. As quantity and price are considered to be fixed in the derivation of this WTP, it is implicitly assumed that the agents do not move to other locations in response to the change in quality. While this assumption can be maintained in case of marginal changes, a WTP for non-marginal changes requires the specification of both demand and supply and the introduction of endogenous prices. Since the WTP in hedonic pricing is consistent with a Hicksian consumer surplus, locational sorting models introduce a WTP that is adjusted for price changes, the ‘general equilibrium willingness to pay’ (GE-WTP). Section 7.2 argued that ‘general equilibrium’ corresponds to a spatial equilibrium in urban economics. Therefore, a GE-WTP could also be read directly as a monetary measure for a change in *spatial welfare*.

The supply of housing in locational sorting models is often taken as fixed, assuming that the population will resort over the existing stock of houses. The model of chapter 6 can be used for a consistent definition of a GE-WTP with an endogenous number of locations, a feature adopted from the models in urban economics. Section 7.3 showed that this same feature also allows for an assessment of the efficiency of the distribution of land. If a simple type of agglomeration externality is introduced, the allocation is no longer efficient. Although this observation is consistent with the neoclassical perspective on external effects, an interpretation of agglomeration externalities as endogenous amenities is preferred. A GE-WTP is then a welfare measure that accounts for the capitalisation of the value of local quality, exogenous or endogenous.

Section 7.4 applied the concept of locational sorting to local marginal changes. It showed that the ratio of the coefficients for quality and space in the preference structure can be used for identifying the location that yields the highest marginal increase of social welfare for the entire population, following a marginal local quality improvement. It suggested that if space is preferred over quality, improvements at the city centre will have limited effects on reducing urban sprawl.

Finally, section 7.5 introduced the concept of a GE-WTA as a measure for the value of open space. In theory it can capture the willingness to accept using less space for residential purposes. Converted to social opportunity costs for land, the virtual rent received as a measure for the value of open space as a pure public good was proven to exceed the market rent for residential use of the same location.

7.7 Appendix

7.7.1 Equivalent problem for the social planner

Section 7.1 stated that the maximisation problem for the benevolent social planner,

$$\max_{\mathbf{s},\mathbf{z},M} \sum_{j=1}^{M} n_j u_j \left(s_j, z\right) \quad \text{s.t} \quad Ny = Nz + \sum_{j=1}^{M} p_j A, \tag{7.7.1}$$

is essentially the same as

$$\max_{\hat{S},Z} \hat{S}^{\beta} Z^{1-\beta} \quad \text{s.t} \quad Y = Z + \sum_{j=1}^{M} \left(\frac{p_j}{q_j^{\frac{\gamma}{\beta}}} \right) \left(A q_j^{\frac{\gamma}{\beta}} \right), \tag{7.7.2}$$

where

$$\hat{S} = \sum_{j=1}^{M} \hat{S}_j = \sum_{j=1}^{M} \left(A q_j^{\frac{\gamma}{\beta}} \right). \tag{7.7.3}$$

The equivalence follows from lacking a degree of freedom in maximising $\hat{S}$ because of the supply constraint per location. Therefore,

$$\begin{aligned}
\sum_{j=1}^{M} n_j u_j &= N \sum_{j=1}^{M} x_j u_j \\
&= N \sum_{j=1}^{M} s_j^{\beta} z^{1-\beta} q_j^{\gamma} \\
&= N z^{1-\beta} \left(\frac{A}{N}\right)^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} q_j^{\gamma} \\
&= Z^{1-\beta} A^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} q_j^{\gamma}. \qquad (7.7.4)
\end{aligned}$$

Next, from

$$A^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} \left(q_j^{\frac{\gamma}{\beta}}\right)^{\beta} = \left(\sum_{j=1}^{M} A q_j^{\frac{\gamma}{\beta}}\right)^{\beta}, \qquad (7.7.5)$$

it follows that

$$\sum_{j=1}^{M} x_j^{1-\beta} \frac{\left(q_j^{\frac{\gamma}{\beta}}\right)^{\beta}}{\left(\sum_{j=1}^{M} q_j^{\frac{\gamma}{\beta}}\right)^{\beta}} = 1. \qquad (7.7.6)$$

And finally, given $\sum_{j=1}^{M} x_j = 1$ the solution is given by

$$x_j = \frac{q_j^{\frac{\gamma}{\beta}}}{\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}}}. \qquad (7.7.7)$$

7.7.2 Simplified expression for plotting figure 7.6

In this subsection of the appendix the expressions that helped simplifying (7.5.7) plotted in figure 7.6 are derived.

$$\begin{aligned} x_{M+1} &= \frac{q_{M+1}^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M+1} q_k^{\gamma/(\beta+\mu)}} \frac{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}} \\ &= \frac{q_{M+1}^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}} \left(1 - x_{M+1}\right). \end{aligned} \tag{7.7.8}$$

$$\begin{aligned} \frac{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M+1} q_k^{\gamma/(\beta+\mu)}} &= \frac{\sum_{k=1}^{M+1} q_k^{\gamma/(\beta+\mu)} - q_{M+1}^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M+1} q_k^{\gamma/(\beta+\mu)}} \\ &= \left(1 - x_{M+1}\right). \end{aligned} \tag{7.7.9}$$

$$\frac{x_{M+1}}{\left(1 - x_{M+1}\right)} = \frac{q_{M+1}^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}. \tag{7.7.10}$$

$$\frac{q_{M+1}^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}} + 1 = \frac{1}{\left(1 - x_{M+1}\right)}. \tag{7.7.11}$$

Chapter 8

An MAS approach to discrete locations choices with interactions

8.1 Introduction

This chapter develops a model that contains several extensions of the model presented in chapter 6. Furthermore, it will be implemented as a computational model that conforms to the specification of a *multi-agent system* (MAS). In line with the previous chapters, the attempt to find a balance between the benefits from an agent-based approach in terms of behavioural interpretation of decisions made by individuals and the normative interpretation from the traditional neoclassical framework and game theory, guides the strategy of constructing the model.

Although the behavioural rules of the two-agent model developed in chapter 4 were relatively simple, special attention was given to the more cognitive interpretation of information processing capabilities of the individual agents in the bargaining variant. The only information the two agents exchanged consisted of proposed price-quantity pairs. No further information regarding the other agent's characteristics or preference structure was required for the agents to make their decisions. Occasionally, the term 'agent-based model' is reserved for this type of model, consisting of a few agents equipped with relatively advanced behaviour rules. In contrast, a 'multi-agent system' consists of a large number of agents with rather simple behavioural rules (Axtell, 2000). In general a MAS focuses more on the patterns that emerge from the interactions within a large population of simple agents. It can conform to the identification of the interactions at the levels individuals as the origin of complexity in a complex dynamical system. This chapter focuses on the formation of land use patterns using an MAS. The implementation of the model as an MAS allows in principle for the interpretation as an *artificial society* (Epstein and Axtell, 1996). However, since the behaviour of the agents is still relatively abstract, the interpretation as a *swarm* or *particle system* can be applied as well. The model in this chapter serves as an example of a system consisting of a large collection of semi-autonomous agents organising itself in spatial patterns, relying only on simple decisions rules. In line with the evolutionary game theoretical models in chapter 6, the economic interpretation is supported by the fact that the agents act myopically only in their self-interest. The stationary spatial—and market—equilibrium can be considered a Nash equilibrium.

Chapters 1 and 5 discussed the literature stressing the relation between discrete choice, evolutionary game theory, and statistical physics. Literature in which the interpretation of a dynamical discrete choice model as a simplified multi-agent model is employed also exists (see for example Brock, 1997; Brock and Hommes, 1997). Finally, regarding the application of an MAS to land use modelling, related models can be found in Page (1999) and Otter et al. (2001), but they do not account for price formation on a land market. The *implementation* of a *discrete choice model* with interactions *as an MAS*, is—to the knowledge of the author—an addition to the existing literature. Consequently, the combination of discrete choice and an MAS in a single framework will be labelled *Multi-Agent Discrete Choice Model* (MADCM) throughout this chapter.

With an MADCM land use model, two types of computational experiments will be conducted. The first is a special type of experiment that can only be conducted with an MAS. This concerns the relaxation of the *mean-field* assumption of the population model. Once agents can be distinguished individually, it is possible to restrict the information they receive. The compact representation of the replicator dynamics and related evolutionary dynamical systems usually comes at the price of the assumption that each individual has a probability of meeting all other individuals. With the discrete choice variant and interaction through disequilibrium prices, this assumption can be translated to *perfect information*. It is implicitly assumed that the agents will move to the best available location based on the information regarding the prices, quality levels, and population densities for *all* locations. With an MADCM, this information can be restricted to a reduced number of—possibly random—locations. Experiments with imperfect information can shed light on the robustness and stability of certain agglomeration patterns.

The second experiment concerns a more applied interpretation of an extension of the model in section 7.5, regarding open space. This experiment addresses the distributional effects, if open space is—in addition to a pure public goods—also considered a source of positive external effects, affecting quality level and the land price of the locations next to the open space. Only higher income groups are expected to be able to afford living at these locations. The policy question arises whether the GE-WTA the higher income group is supposed to pay for a smaller agglomeration in return for open space is relatively higher or lower than the GE-WTA for the lower income group.

This chapter is organised as follows. Section 8.2 presents a more detailed description of the MADCM developed in this chapter, with a focus on land use in section 8.3. Section 8.4 presents two extreme cases. The first concerns an extension of the Alonso variant of chapter 6, representing a land use model without external effects. The second is a variant of Schelling's segregation model (Schelling, 1978) by means of an example of a land use model based only on external effects. Section 8.5 devotes itself to a variant of a model originally developed by Beckmann (1976), which can be interpreted as an Alonso-type of model with an endogenous central business district (CBD). Section 8.6 presents the results from a computationally oriented experiment with imperfect information. Finally, section 8.7 discusses a more policy motivated simulation run, exploring the equity implications of open space externalities. This chapter finishes with a discussion and conclusions drawn in section 8.8.

8.2 MADCM: general concepts

An MADCM is based on several concepts that do not depend on the specific interpretation in the context of land use. They are presented in this section, while the more land use related issues are discussed in section 8.3.

8.2.1 Model construction

The Multi-Agent Discrete Choice Model (MADCM) is based on the discrete choice formulation as a stochastic population game and its relation with the CES preference structure for a representative consumer, discussed in chapter 6. The starting point is the dynamics of equation (6.4.12) from section 6.4 repeated here:

$$\dot{x}_j = \frac{\left(p_j \Big/ q_j^{\gamma/\beta}\right)^{-\beta/\mu}}{\sum_{k=1}^{M} \left(p_k \Big/ q_k^{\gamma/\beta}\right)^{-\beta/\mu}} - x_j. \tag{8.2.1}$$

Since the first term on the right-hand side of (8.2.1) is identical to the logit model, the dynamics can be interpreted directly in terms of a behavioural rule based on the cumulative probability function (6.4.5),

$$\Pr\left(\ln v_{i,j} - \ln v_{i,k} > -\mu\varepsilon_{i,jk}\right) = \frac{v_j^{1/\mu}}{\sum_{k=1}^{M} v_k^{1/\mu}}. \tag{8.2.2}$$

Here, the stochastic term, $\varepsilon_{i,jk} = \varepsilon_{ij} - \varepsilon_{ik}$, consists of the difference between the idiosyncratic elements in the preference structure of agent i regarding locations j and k. The model of (8.2.1), however, still relies on a representation of agents as a *continuum*. The computation of the equilibrium solution for (8.2.1) can be performed as a numerical integration of a system of non-linear ordinary differential equations. The interpretation of the model as a stochastic population game is based on the equivalence of a probability density distribution for the location choices of a population of agents with identical preference structures—except for the stochastic term $\varepsilon_{i,jk}$—and the density distribution of the population. This interpretation amounts to the assumption that the population is so large that it can be considered a continuum. As a result of the immense population size, the idiosyncratic components of the preference structures form a continuum as well, because every value for $\varepsilon_{i,jk}$ is represented by some agent i.

Alternatively, instead of a distribution of the population density, the stationary equilibrium for (8.2.1) can be considered a probability distribution for the location choice of the *individual* agent. This interpretation conforms to the identification of the logit choice model (8.2.2) as the solution ($\dot{x}_j = 0$) for (8.2.1) for all j. It also corresponds to the way the model in this chapter is constructed. A large but limited number of N agents are implemented as N objects and each agent is equipped with an individual decision rule, with (8.2.2) as the basis for a condition as in chapter 4. The corresponding conditional[1] indirect utility function can be written as

$$\ln v_{ij} = \ln(\alpha y_i) - \beta_i \ln p_j + \gamma_i \ln q_j, \tag{8.2.3}$$

but variants of (8.2.3) can be used as well, as presented in the following sections.

With (8.2.1) interpreted as a type of algorithm for finding the cumulative probability density (8.2.2) as $F(\varepsilon)$, the population game can be considered a special way of integrating a probability density function, $f(\varepsilon)$, as in

$$F(\varepsilon) = \int\limits_{-\infty}^{\infty} f(\varepsilon)\, d\varepsilon. \tag{8.2.4}$$

Integration by means of an evolutionary selection mechanism is necessary for (8.2.3), because the equilibrium price vector, $\mathbf{p}(\mathbf{x})$ depends on the demands—as the vector of population densities, $\mathbf{x}$—at all locations. In this way, the Nash equilibrium that results from integrating (8.2.1), can be considered an example of *self-consistency* that in as a static solution would conform to the definition of *rational expectations* (Brock and Durlauf, 2001). Self-consistency refers to the solution in which the individual expectations concerning the population distribution matches the realised distribution. Consistent with the original concepts from evolutionary game theory, self-consistency emerges from a process that requires only a type of bounded rationality in the models developed in this thesis.

The analogy with (8.2.4) also allows for an alternative integration algorithm. An MADCM can be interpreted as an MAS implementation of (8.2.1) by means of a type of *Monte Carlo integration* (Judd, 1998), inspired by the use of simulation methods for discrete choice models (Train, 2003). In an MADCM, the stochastic terms in the individual preference structure will be replaced by the values of actual *draws* from the distribution of this term. This means that every agent i is assigned values from the double exponential distribution of ε_{ij} for all locations j. Whereas this stochastic term was interpreted in chapter 4 as the ‘circumstances’ that affected the choice of a single agent at a given moment in time, here they are interpreted as an additional motivation for the individual i for preferring location j. This term is supposed to be known by the agent, but unknown to the observer (see also Fudenberg and Levine, 1998, p. 106). Draws are taken once for preference structures of all agents, which requires $M \times N$ draws from a double exponential distribution.

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[1] The utility function reflects the value of the level of utility for the individual agent, i, *conditional* upon her location of choice being location j.

In this way, the local population density can be expressed as the number of agents at a given location, divided by the population size. The density can be considered as an approximation of the logit model. This approach is similar to the approximation of the logit model as a time average, discussed in section 4.3.2. With an evaluation function, $I_{ij}\left(\bullet\right)$ that has a value of 1 if agent i chooses location j and a value of 0 otherwise, the population density at location j, for a population consisting of N agents, can be written as

$$\hat{x}_j = \frac{1}{N}\sum_{i=1}^{N} I_{ij}\left(\ln v_{ij} + \mu_i\varepsilon_{ij} = \max\left\{\ln v_{i0} + \mu_i\varepsilon_{i0}, ..., \ln v_{iM} + \mu_i\varepsilon_{iM}\right\}\right). \tag{8.2.5}$$

In 8.2.5 the evaluation function is applied to verify whether the location j yields the *maximum* level of utility for agent i, taking into account the idiosyncratic components, ε_{ik}, for all M locations. From (8.2.1) it follows that the agent selects the location that yields the highest level for $\ln v_k + \mu_i\varepsilon_{ik}$. Although v_k is essentially an indirect utility function, the model allows—as in chapter 4—for an interpretation in terms of current relative price differences that need to exceed a threshold defined by relative quality differences, before agents choose a different strategy. In this case maximisation is identical with a comparison of the utility level at the current location with the levels at all other locations, possibly followed by a move to the current best location.

Unlike the usual integration methods of the Monte Carlo type, however, interaction terms need to be accounted for, because the price, p_j, is endogenous since it depends on the location choices of all other agents, as noted above. Since this dependency can be related to the population densities at the other locations, $v_{ij} = v_{ij}\left(\mathbf{x}\right)$, a case of *strategic interaction* results, similar to that in a population game. The implementation of the evolutionary selection mechanism according to section 6.4.1 is in this perspective basically an *iteration* in terms of a numerical calculation. In an MADCM this iteration is supported by an MAS-like behavioural interpretation. With respect to simulation methods in econometrics, every simulation run could be considered as one possible realisation[2].

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[2] If an MADCM is to be applied as a real Monte Carlo integration, a very large number of simulation runs should be executed, using $M \times N$ new draws for every run. The researcher should keep track of the individual agents, followed by the interpretation of the averages individual equilibrium location choices as an estimation of the probability distribution for the agent concerned.

8.2.2 Platform

The MADCM has been implemented as a software tool developed by the author. There is a variety of programmes and libraries of functions for a modeller building an MAS or an ABM. Popular examples are *RePast* and *SWARM*. In principle, the MADCM could be implemented using any of these existing platforms. The choice not to do so was guided by the goal of integrating an MAS with a more traditional approach, inspired by Monte Carlo integration, described above. The possibility of incorporating an existing high-performance library with numerical routines—for the generation of random numbers and the manipulation of matrices—was considered more important, than tools that facilitate the implementation of agents.

The final version was written in the language C++ (Stroustrup, 2000). The library used is the *GNU Scientific Library* (GSL), version 1.8. This is a library of functions in *C* (Kernighan and Ritchie, 1983), published as *open source* under the *GNU Public License* (GPL). The architecture of the agents and the populations is relatively simple and can be implemented in any object-oriented language such as *Java* or *C++*. Since C++ can be considered an extension of the C-language, C++ was the obvious choice for combining the fast and high-quality numerical algorithms of the GSL with an agent-based architecture.

To facilitate making runs with different parameter values, a function written in the language *Python* (version 2.5) was integrated in the programme. Python is a scripting language originally written in C. The benefit of a scripting language is that a programme does not need to be *compiled*—that is, translated to machine code[3]—first before it can be run. The disadvantage of a scripted language is, in general, a lack of performance (speed) compared to a programme written in a compiled language. Agents in the MADCM receive their preference structure at initialisation by passing parameter values in a small *embedded* script. After initialisation, the benefit of the compiled language C++ is exploited for the actual runs, consisting of a large sequence of changing location choices[4].

Different versions of the MADCM, were compiled with two different compilers:

1. the *GNU Compiler Collection* (GCC) was used under both Linux and the *Cygwin* UNIX emulator for Microsoft Windows,
2. the *MS Visual C++* compiler was used in the ‘Visual C++ 2005 Express’ edition.

Standard platforms for building agent-based models usually have a relatively large library of functions for building a graphical user interface (GUI). Since the GUI of

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[3] See also the discussion on machine code and Turing Machines in chapter 2.

[4] This combination of scripts and a compiled main programme resembles the integration of scripting languages in, for example, computer games.

the MAS only served for the purpose of inspection while making runs, only a simple GUI was designed. Use was made of a small set of function from *OpenGL* library. OpenGL is also written in C and was originally developed for high-performance 2D and 3D graphics. Integration of OpenGL in a windowing environment was accomplished by the *Qt* library under Linux and Cygwin. The *GLUT* library was used for the MS Windows versions. The translation of the output to some of the graphs in this thesis was performed by a script, written by the author in Python that uses data files as input and returns a LaTeX-file as output. In the LaTeX-file, use was made of the *pstricks* package.

8.3 MADCM land use model

The implementation of the model from chapter 6 as an MADCM is motivated by the need of exploring a wider range of external effects. In chapter 7 only a variant of *network* externalities was introduced. It served as a simple agglomeration externality, or *communication externality* in the terminology of urban economics. Only the population density at a current location contributed to the level of well-being of the individual agent. Agglomeration forces that can help explaining the emergence of a city require that the level of well-being at one location depends on the population densities at other locations. In line with chapter 7 it will be argued that from a conceptual point of view, these extended externalities can, in the context of land use, best be treated as endogenous amenities or quality levels. Internalisation of the welfare contribution from these externalities as traditional external effects in a neoclassical context is of limited use for policy purposes. Rather, the welfare effects of capitalisation can be accounted for, as would be done in case of exogenous amenities in urban economics and hedonic pricing. Once this interpretation is adopted, a nearly unlimited amount of endogenous amenities can be defined, including complex, nonlinear ones. An MADCM is especially well-suited for exploring the effect of highly non-linear endogenous amenities on the self-organising system, the resulting land use patterns, and their welfare effects.

The use of an MADCM as a land use model requires an account of a number of specific elements. These elements include a two-dimensional grid, the implementation of agglomeration externalities, and the interpretation of disequilibrium land use patterns.

8.3.1 Grid

The model presented in chapters 6 and 7 was a stylised model in one dimension. In the land use MADCM, a grid consisting of 896 hexagons will be used to extend the model in two dimensions, as plotted in figure 8.1. The choice for hexagons is essentially arbitrary, but is expected to facilitate the interpretation of circular patterns. The grid can be interpreted as all the land available in a certain region. In some agent-based models the grid is wrapped and folded into a ‘doughnut’ shape (Epstein and Axtell, 1996). This is usually done if only *local interactions* are important and complications at the borders of a grid need to be avoided. Given the explicit spatial interpretation of the grid in the MADCM in this chapter, the grid will not be wrapped. The borders are considered the borders of the region. Complications are not expected, as agents are either expected to locate in or nearby an agglomeration. If necessary, the borders can be interpreted as natural barriers. For the distance between the locations an 896 by 896 *distance matrix* is calculated. Depending on the type of simulation run, this matrix might play a role in the preference structure of the agents (see section 8.3.2).

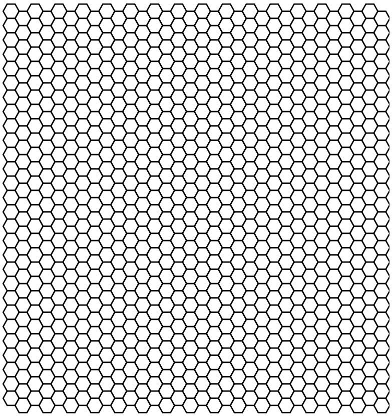

Figure 8.1: *Empty grid for MADCM land use model.*

At the beginning of each run, a population of agents is distributed randomly over the grid, illustrated by figure 8.2. The colour of a grid cell reflects the local population density. The darker the colour, the higher the density[5].

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[5] To facilitate the visual inspection of the runs, the colour spectrum is scaled, using the highest and lowest density. The saturation therefore reflects the *relative* differences in density.

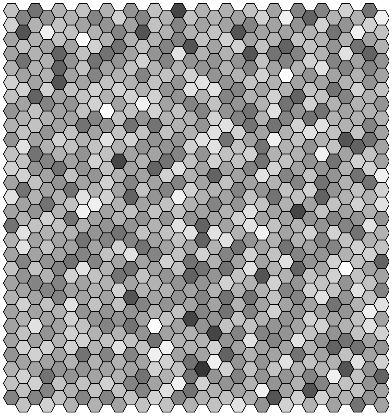

Figure 8.2: *MADCM land use model with random population densities.*

8.3.2 External effects

The equilibrium solution for most models in previous chapters was consistent with the static solutions for neoclassical counterparts in urban economics, due to the absence of external effects. By consequence, they can in principle be restated in a neoclassical fashion and the unique equilibrium will be Pareto optimal. As noticed in chapters 1 and 5, the optimal solution comes at the price of explaining the existence of an agglomeration by means of an exogenous parameter. Most frequently this parameter is the distance to the CBD, usually captured in transportation costs. In chapter 6 this parameter was translated to an exogenous parameter indicating the local quality—or amenity—level.

Completely in line with traditional welfare economics, a translation is thereby made from a public good to an externality by making the local quality *endogenous*. Here, endogenous means that the quality depends on the behaviour of other agents. Since the local quality affects the level of well-being of an agent residing at that location, a direct impact on the level of well-being without intermediation through prices is an *external effect*. Chapter 4 showed that external effects might give rise to multiple equilibrium solutions by introducing a degree of interdependence in the preference structure of agents. In general it can be argued that externalities give rise to more *complexity*, in the sense of complex dynamical systems, since self-organisation will be complemented with path dependency (see also section 3.4). With the evolutionary dynamics introduced in the previous chapter, this type of complexity will be explored further by adding an endogenous quality parameter to the MADCM.

This chapter extends the exogenous quality parameter by a parameter that reflects an endogenous local quality, that in principle can depend on the population densities at all locations. The resulting logarithm of indirect utility function plus idiosyncratic term for agent i at location j can be written as

$$\ln v_{ij} + \mu_i \varepsilon_{ij} = \ln (\alpha y_i) - \beta_i \ln p_j (\mathbf{x}) + \gamma_i \ln q_j + \delta_i \ln f_{ij} (\mathbf{x}) + \mu_i \varepsilon_{ij}. \quad (8.3.1)$$

In (8.3.1), the level of indirect utility depends on the fraction, α, of the income, y, the agent spends on land, the endogenous price, p_j, the exogenous quality level, q_j, and the location-specific—and possibly individual-specific—function $f_{ij} (\mathbf{x})$ that defines the endogenous quality level.

Section 6.5 introduced some level of heterogeneity by extending the single population model to a model with four populations, distinguished by income and an income-dependent preference structure. This concept can also be applied to all coefficients in the MADCM, including

$$\begin{aligned} \beta_i &= \beta_1 + \beta_2 \ln y_i \\ \gamma_i &= \gamma_1 + \gamma_2 \ln y_i \\ \delta_i &= \delta_1 + \delta_2 \ln y_i. \end{aligned} \quad (8.3.2)$$

Although in principle every individual can be assigned a different coefficient, this chapter uses only a differentiation of the population according to sub-populations, where individuals in a sub-population have the same coefficients.

8.3.3 Interpretation of the disequilibrium configurations

The MADCM can be analysed from a system's perspective, based on the derivation of the basic mechanism determining how the land market operates in chapter 6. The final land use pattern is interpreted as the stationary state of what is in principle an economy in disequilibrium. Disequilibrium land prices are quoted as the price per surface area based on the total amount of money offered. This means that in the process of finding a spatial equilibrium, land prices depend on the consumers' willingness to pay to achieve the equilibrium utility level. Location choices at disequilibrium prices determine the size of the surface area rented as the average amount of space per agent. Given the current price, the agent can decide whether the amount of money spent on land at the current location corresponds to her actual willingness to pay. As a result, both the price and the equilibrium utility level depend on the location choices of all other agents.

Two interpretations were possible for the disequilibrium configuration of the model in chapter 6. In line with the two-agent model developed in chapter 4, in the first interpretation the location choices out of equilibrium might be interpreted as an ongoing negotiation process in which no agent moves before an agreement is reached. In this interpretation price formation out of equilibrium can be regarded best as a process in which bid rents are collected by a broker—the land owner or a representative—for every location, acting as a local myopic auctioneer for the location. Next, the brokers determine and announce the new prices. As a result, price setting is part of a feedback mechanism and the individual location choices can be considered low level elements in high level complex adaptive system. It would suggest that no agent moves until the equilibrium prices for all locations are established in a process of collective negotiation. Although this interpretation is suitable in the context of bargaining with two agents, it does not supply a very realistic picture in the case of many agents.

The second interpretation of the model in chapter 6, is also the only feasible interpretation of the MAS variant in this chapter. It is assumed that the agents actually choose a location. Prices are then determined in the same way described above. However, since locations are chosen, these prices are disequilibrium prices only in the sense that the agents appear to be dissatisfied with their current location choice. If they receive an opportunity to move to a better location, they will. The choice to move is myopic, since it is assumed that the agent determines the level of utility for all—or several (see section 8.4)—locations on the basis of the current situation, not taking into account the possible effect of his own move. Although the entire process might not give an accurate description of the development over time of a real agglomeration, it does not pose any unrealistic assumption on the cognitive capacities of the individual agent. In this perspective, the emergence of an agglomeration in this chapter is perceived as an abstraction of an evolutionary process. The behaviour of the individual can still be considered as more or less realistic, in the sense that he only acts in his self-interest while improving only his own location choice, without being concerned about the overall development of the agglomeration or its effect on the social welfare level of the population as a whole.

8.4 Simulation runs with externalities

To illustrate the flexibility of the MADCM and the effect of externalities, the model is adapted to encompass three existing models. First, the variant of the Alonso model developed in chapter 6 is be extended to the two dimensions grid and implemented with agents. This is an example of a model with only exogenous amenities. Next, a model with only endogenous amenities is presented; a variant of the Schelling model (Schelling, 1978). Finally, the two extremes are combined in one model.

All models in this section consist of 10,000 agents. In the first three examples, the population is divided into two groups of equal size. The two groups are distinguished by a different income level and the preference structure is income-dependent according to (8.3.2).

8.4.1 Alonso

The first model uses the distance matrix described in section 8.3.1. One cell in the middle of the grid is assigned a function similar to that of a CBD. All other cells receive a quality level that depends on the distance to the cell in the middle. The agents have a preference structure that includes an impact on their level of well-being of the quality level and the endogenous price at the particular grid cell they choose. In terms of (8.3.1) and (8.3.2), $\delta_i = 0$, because only prices and exogenous quality levels determine the location choices. To maintain a close reference with the original model, by setting $\mu_i = 0$, the stochastic term does not have an impact on the preference structure of the agents.

Simulation results are plotted in figure 8.3, with one plot for each of the two sub-populations. The well-known concentric circles appear. The darker the grid cell, the higher the population density. In the white cells, no agents are located, as a result of opportunity costs from alternative land use. The two sub-populations are segregated by the endogenous prices of the locations, similar to the sorting model presented in section 6.5. The population on the right-hand side earns a higher income and the coefficients of the preference structure give rise to endogenous prices near the CBD

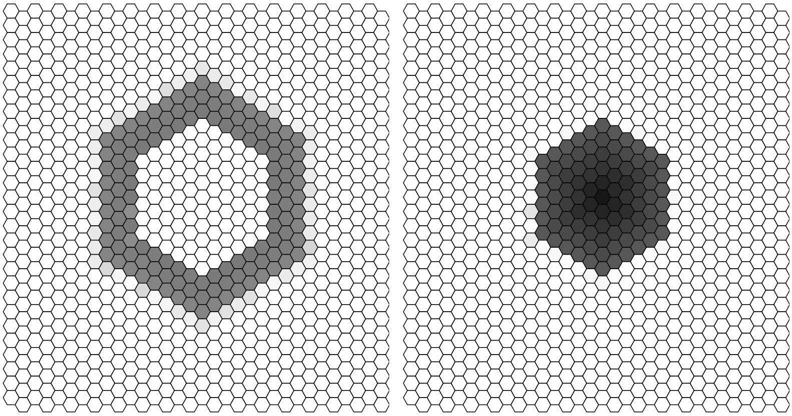

Figure 8.3: *MADCM Alonso variant.*

only this group can afford.

8.4.2 Schelling

The second model is inspired by the segregation model of Schelling (1978, p. 137–166). Here, quality is defined as dependent on the number of neighbouring cells in which agents of the same sub-population are located. In terms of (8.3.1), in this model $\gamma_i = 0$ and $\delta > 0$, since only the endogenous quality level plays a role. Both groups have the same income level. In the original model (Schelling, 1978), a grid cell was occupied by a single agent and there were no linkages to a land market. To stress the complementarity with the Alonso model in the MACDM here, $\beta > 0$, implying that the final land pattern is the result of a balance between endogenous prices and endogenous quality levels. The endogenous quality level is defined as

$$f_{ij}(\mathbf{x}) = \begin{cases} 1 & if \quad n_{ij} > 4 \\ 2 & if \quad n_{ij} \leqslant 4 \end{cases} \tag{8.4.1}$$

In expression (8.4.1), the parameter n_{ij} represents the number of locations that have at least one agent of the same group as agent i. Only the six surrounding grid cells are counted.

In figures 8.4-8.6 the equilibrium patterns for the two sub-populations are plotted separately for three runs with different initial distributions. As expected, there is no

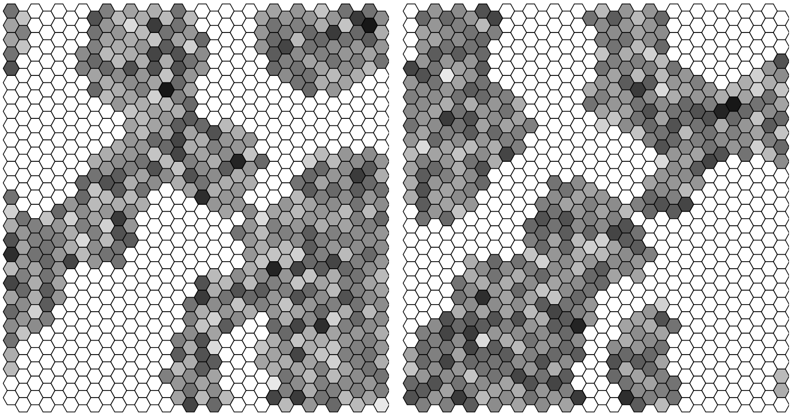

Figure 8.4: *MADCM Schelling variant; run 1.*

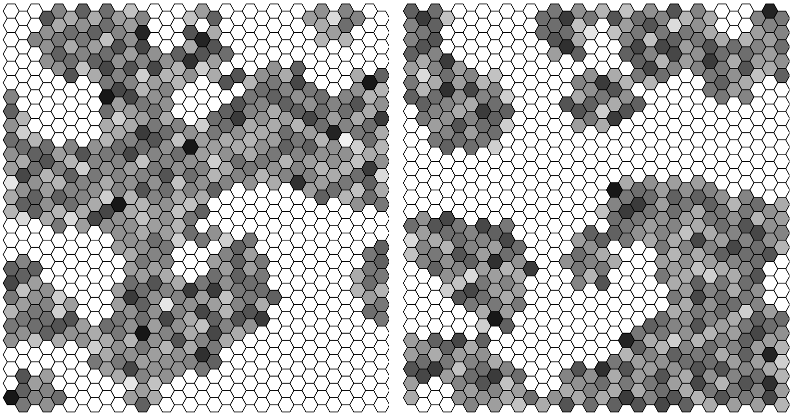

Figure 8.5: *MADCM Schelling variant; run 2.*

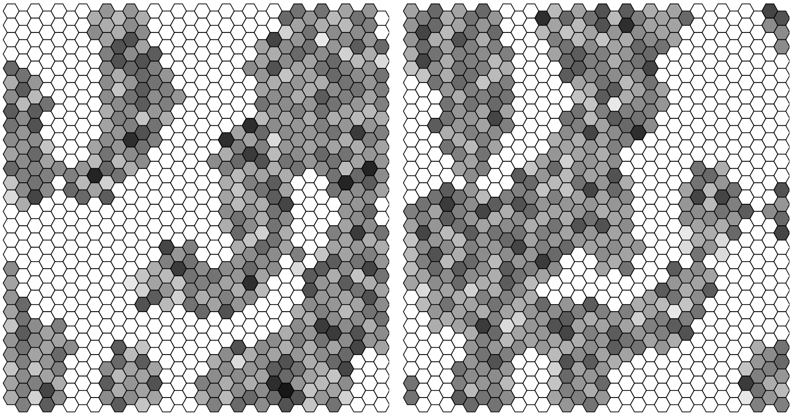

Figure 8.6: *MADCM Schelling variant; run 3.*

notion of a CBD. Instead, clusters of connected grid cells emerge. Schelling's original model served as an illustration of endogenous segregation. Even if agents were not categorically averse to living next to an agent of a different group, neighbourhoods were still strictly segregated according to group characteristics. This effect is also present in figures 8.4-8.6, as the land use patterns of the two populations seem to complement each other. Schelling's model is considered one of the first models of social interactions (Durlauf and Young, 2001). It also counts as one of the first agent-based models (Epstein and Axtell, 1996), even though it was originally implemented with the use of coins and a sheet of paper (Schelling, 1978, p. 147).

In the computational experiment presented here, the model primarily serves as an example of a type of endogenous agglomeration that is enforced by a complex externality. The figures show, that this model also represents an extreme case of multiple equilibria. Only the qualitative result of segregation and agglomeration is obtained; the actual positions of the separate clusters are not unique.

8.4.3 Hybrid

Next, a hybrid model that consists of a combination of the two previous models is presented. In this model, the local quality has an exogenous as well as an endogenous component. Additionally, prices are endogenous, as before. The results of a single run are shown in figure 8.7. Now, the location of the CBD is clearly visible, but segregation no longer appears along concentric circles. Instead, the clusters themselves

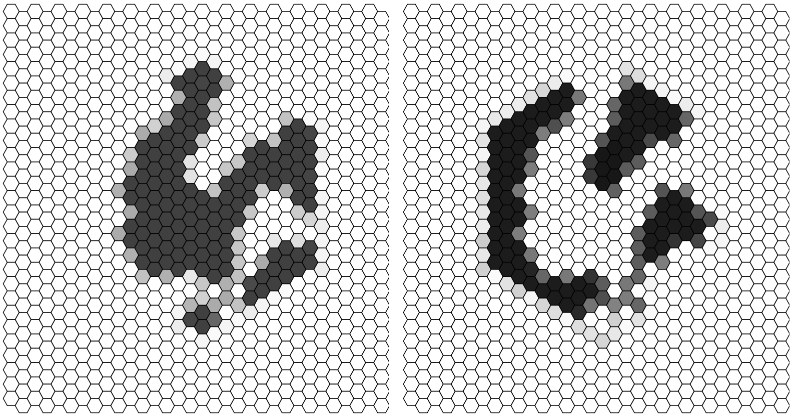

Figure 8.7: *MADCM hybrid (Alonso-Schelling) variant.*

again form an example of multiple equilibria.

8.5 Beckmann

The MADCM can also be transformed into a variant of the endogenous agglomeration model originally developed by Beckmann (1976) and discussed in chapter 5. Here, the quality level is fully endogenous, but—contrary to the Schelling model presented above—the quality level depends not only on the location choices of agents at adjacent locations, but on the population densities at all locations. The endogenous amenity is in this case defined as

$$f_{ij}\left(\mathbf{x}\right) = \tau \sum_{k=1}^{N} d_{jk} n_j = \tau N \sum_{k=1}^{N} d_{jk} x_j, \tag{8.5.1}$$

with d_{jk} as the distance between location j and location k; τ is a constant. Expression (8.5.1) can be interpreted as the average distance to all other agents[6]. Since this average distance has a negative impact on the level of well-being of the individual agent, in (8.3.1) the parameter for the endogenous quality is assigned a negative value, $\delta_i < 0$. The exogenous quality does not play a role in the first two runs, therefore $\gamma_i = 0$. As before, endogenous prices can be considered a centrifugal force[7], hence $\beta_i > 0$.

This model represents a relatively general example of an *endogenous* agglomeration. The land use pattern resembles that of the Alonso model, but instead of assuming the existence of a CBD beforehand, the CBD emerges as a result of social—or *non-market*—interactions. The results from two simulation runs are plotted for a single population of 1000 agents in figure 8.8, for two different values of the parameter, δ_i (0.7 and 0.35), that determines the impact of the social interaction on the well-being of the individual agent, relative to the 'dis-utility' of the endogenous local land price. The parameter controlling the impact of the stochastic term, $\mu_i > 0$, is shown to enhance the dispersal of the agglomeration for smaller values of δ_i. Furthermore, figure 8.8 reveals that the location of the agglomeration is not unique. Because the exogenous local quality does not affect the location choice, only the relative distances determine the final, equilibrium location choices of all agents. This implies that many different equilibria are possible. The presence of multiple equilibria is related to the coordination game discussed in chapter 3. The positive externality from

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[6] In the original model (Beckmann, 1976), and as quoted by Fujita and Thisse (2002), the average distance was incorporated as the cost of travelling to all other agents, analogous to the costs of transportation to the CBD in the Alonso model.

[7] In (8.3.1) the negative impact of the price on the level of well-being is already accounted for by a negative sign.

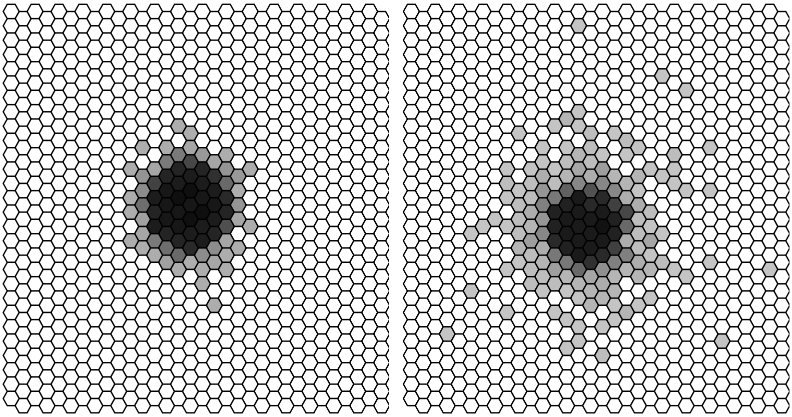

Figure 8.8: *MADCM Beckmann variants with different degrees of dispersion.*

the vicinity of other agents makes the agents indifferent to their absolute position. Similar to the network externality in chapter 4, the quality of the good depends on the actions of other agents.

8.5.1 Lock-in

The occurrence of multiple equilibria in the Beckmann variant of the model in this section can illustrate a possible interpretation of *path dependency* and a *lock-in* situation, discussed in chapter 3, in the context of land use modelling. First, the development of the social welfare function for a general Beckmann model is plotted in figure 8.9. Figure 8.9 shows the evolutionary development of the *sum* of the individual indirect utility levels. In the MADCM implementation of the model, the individual welfare level is in principle a private attribute of the agent. If the agents are forced to reveal their level of well-being to the modeller, the aggregate or sum—based on the relation between the logit model and the model of a representative consumer discussed in chapter 6—can in this way be translated to a *social welfare function* in the MADCM.

Next, the impact of an exogenous quality level is reintroduced ($\gamma_i > 0$) and different exogenous quality levels are assigned to different locations. Some locations are regarded as *obstacles* and were removed from the distance matrix. As a result, these locations were not connected to other locations. In terms of distances, two cells on either side of the obstacles can only be connected through a shortest path

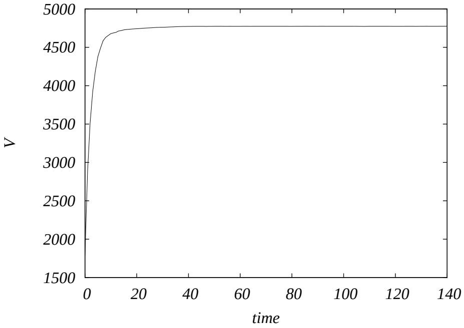

Figure 8.9: *Evolutionary development Beckmann variant.*

around the obstacles. An example of a grid with obstacles is shown in figure 8.10. In addition, the locations above the line of obstacles are assigned a slightly higher exogenous quality level than the locations below the line. This configuration gives rise to two different types of equilibria. The equilibrium agglomeration can emerge above or below the obstacles, depending on the initial random distribution of agents. In figures 8.11 and 8.12, the development of the social welfare function is plotted for

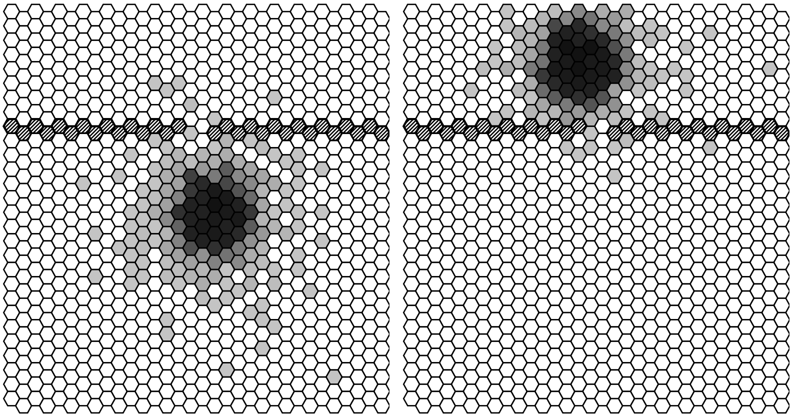

Figure 8.10: *Multiple equilibria: Beckmann variant with obstacles.*

the two solutions of figure 8.10. It shows that the agglomeration above the line of obstacles yields a slightly higher level of social welfare. Therefore, the agglomeration below the line can be considered a *lock-in*, since in theory a Pareto improvement would be possible. The obstacles in this example can be considered a stylised representation of natural obstacles, water for example. The presence of natural obstacles, combined with an evolutionary interpretation of the emergence of agglomerations, illustrate the possibility of a current land use pattern being sub-optimal. A Pareto improvement, however, does not seem feasible in this particular case without rebuilding the entire agglomeration at a new location.

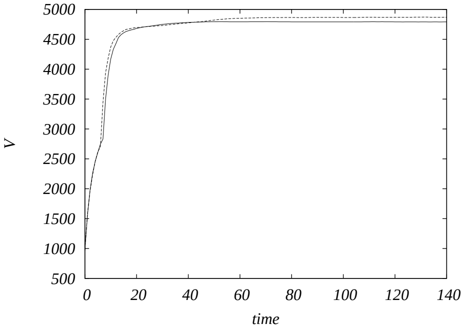

Figure 8.11: *Value of social welfare function for 8.10 (Lock-in).*

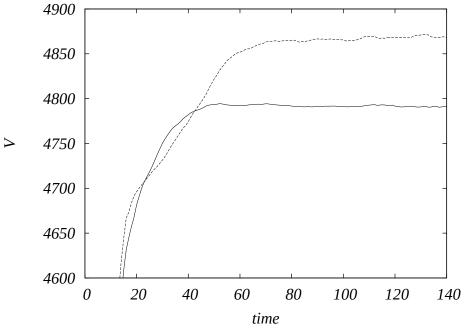

Figure 8.12: *Zoom in on detail of figure 8.11.*

8.6 Imperfect information

The models presented thus far, refer to a situation of disequilibrium that corresponds to an evolutionary path toward the stationary equilibrium solution. Although a stochastic component was introduced in the individual preference structure, this component did not result in a perturbation of the system. This was shown for the formulation in chapter 6, in which the dynamics of the system was essentially deterministic, although location choices were defined in terms of probability densities. In the MADCM variant presented in this chapter, the stochastic term is converted into a fixed draw from a distribution for all individual-location combinations. The stationary equilibrium is stable only in a stochastic sense —as some agents continue to revise their location choices—, but the overall pattern of location choices remains relatively stable and is apparently not under the influence of any notion of randomness.

This section introduces a new type of randomness, more in line with a real perturbation of the system. The main goal of the experiment presented here consists of testing the stability of the equilibrium location pattern against a distortion from within the system. This distortion is introduced by limiting the information available to the individual agent when she is eligible for revising her location choice. As discussed in the introduction, the replicator dynamics and related evolutionary mechanisms abstract from space. In these models it is implicitly assumed that either every agent can meet every other agent, or that the information about all agents is available to all agents. In the model variant presented below, only the information from a limited set of *randomly* chosen locations is made available to the agents during each iteration. This procedure can be thought of as an abstraction of a real relocation process. In reality, if an agent decides to move, she can generally only choose a new location from a limited set of locations, determined by the set of locations where there exists vacancy. In the model presented here no vacancy exists, because it is assumed that the available land at a location is always divided equally among all agents located there. Nevertheless, a restriction of the relocation opportunities, governed by some notion of chance, might be considered a useful test in light of the abstraction of the dynamics already chosen. Instead of trying to account for the real development over time, the concept of evolution is introduced by means of bounded rationality. Here, the convergence of the myopic relocation process is tested against the removal of another stylised, neoclassical assumption: *perfect information*.

The starting point of the simulation experiments is the Beckmann variant introduced in section 8.5, without obstacles. Instead of revealing the information from all 896 locations, the agents receive information from the first 300 locations. If any of these locations would yield a higher indirect utility level, agents move to a randomly picked location. It appears that in this model the process of self-organisation might be seriously hampered, as illustrated by figure 8.13 After an initial attempt to form

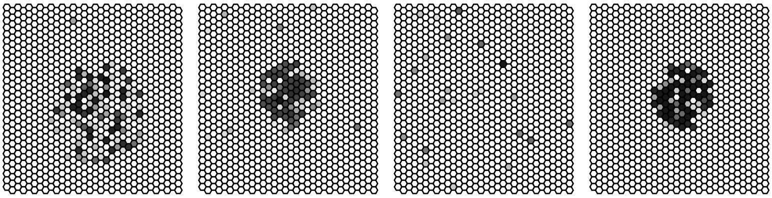

Figure 8.13: *Transient equilibrium configuration with imperfect information.*

an agglomeration, the agents are dispersed again, followed by new attempts, until the system settles in its final agglomeration pattern. The corresponding development of the social welfare function is plotted in figure 8.14. The results for the social welfare

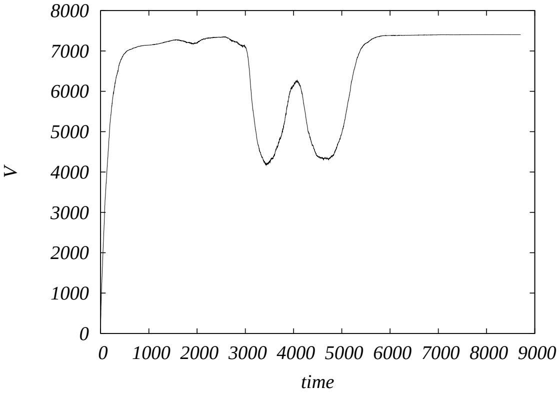

Figure 8.14: *Evolution of social welfare of 8.13.*

function for various initial random distributions, are plotted in figures 8.15-8.16. The main conclusion drawn from these experiments is that although the final stationary equilibrium is the same in all runs, the route toward this equilibrium might contain several transient configurations. In some cases a transient configuration resembles an equilibrium and is continued for some time, before the agglomeration disintegrates again. This conclusion allows for an interpretation that suggests that—in addition to the possibility that a current land use pattern reflects a lock-in, demonstrated in

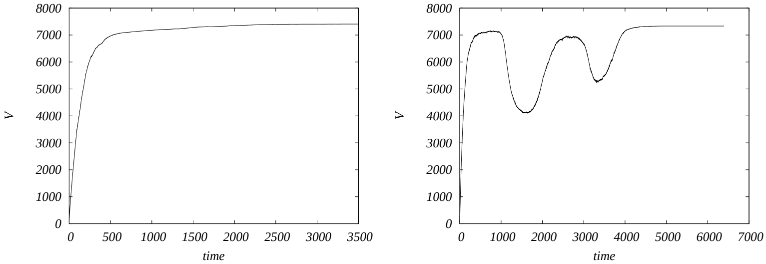

Figure 8.15: *Examples of evolution of social welfare with imperfect information a.*

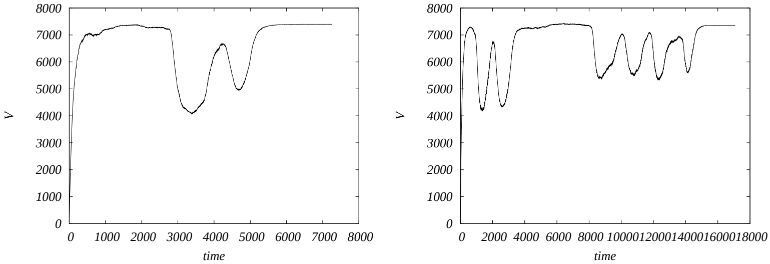

Figure 8.16: *Examples of evolution of social welfare with imperfect information b.*

section 8.5—a current land use pattern might also reflect a temporary, transient and unstable configuration. The policy relevance of this conclusion is primarily that the identification of a given land use pattern with a social optimum has to be made with caution.

8.7 Equity effects of open space

The final experiment serves as an illustration of a more applied and semi-quantitative experiment. It also continues the discussion on the value of open space begun in chapter 7. In section 7.5, open space was essentially considered a pure public good at the aggregate level of the entire population and its presence was assumed to affect the level of well-being of the agents, irrespective the exact location that was supposed to be left undeveloped. This section assumes that open space also affects the local quality level. Similar to the MADCM variant of the Schelling (1978) model of section 8.4, a non-linear endogenous quality level is introduced. Instead of the number of locations in which agents of the same group are located, here the number of empty locations characterise the endogenous amenity.

As a result, the price index, G, is not only affected by the number of developed locations, but also by the endogenous quality level. The result is a relatively complicated feedback mechanism that determines the value of the social welfare function. This mechanism is depicted in figure 8.17 as a conceptual scheme.

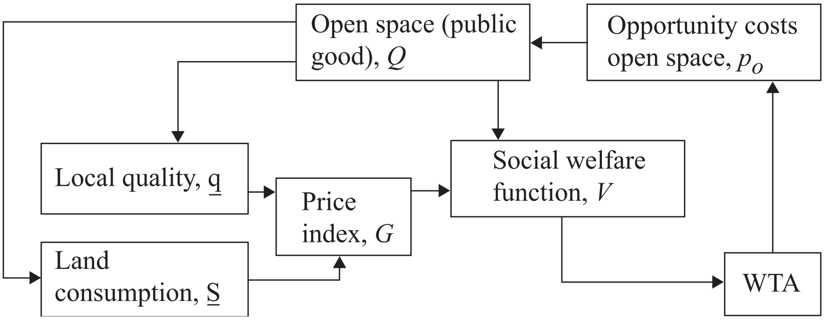

Figure 8.17: *Conceptual representation of the spatial welfare function*

First, the concept of a general equilibrium willingness to accept (GE-WTA) that corresponds to the value of open space is repeated in a simplified fashion. The social welfare function for the representative consumer can be written as

$$V = YG^{-\beta}. \tag{8.7.1}$$

Since all impacts—prices, quality levels and number of developed locations—are captured in the price index, the GE-WTA for a change in the equilibrium land use pattern can be derived from

$$YG^{-\beta} = \left(Y + N \cdot WTA_{GE}\right)\hat{G}^{-\beta}. \tag{8.7.2}$$

This assumes that $\hat{V} < V$, implying that the level of welfare after the change is lower than before. This is consistent with the *reduction* in the number of developed locations when open space is created. From (8.7.2) it follows that the GE-WTA can be written as

$$N \cdot WTA_{GE} = \left[\left(V - \hat{V}\right)\Big/\hat{V}\right]Y. \tag{8.7.3}$$

Regarding the MADCM, (8.7.3) implies that the GE-WTA can be approximated if the value of the social welfare function—in the MADCM simply the sum of the individual welfare levels—is known. If a population is divided into several income groups, each group has its own equilibrium welfare level. This follows from the observation that in equilibrium no agent has an incentive to move. Because agents within a group are identical (up to the idiosyncratic component in the utility function), this also applies to the individuals within a group. Consequently, a GE-WTA for each group can be calculated—based on (8.7.3)—, using an MADCM.

A population of 10,000 agents is divided into four groups for the experiment. The starting point is the Alonso variant, presented in section 8.4.1. The income-dependent preference structure results in the one-dimensional distribution plotted in figure 8.18. In line with the sorting examples presented in chapter 6, the groups with the higher incomes are located near the CBD. Groups with lower incomes live in the ‘suburbs’, having a smaller lot size per agent than the groups with the higher incomes. The land use patterns of the corresponding MADCM in two dimensions are plotted for the respective groups in figure 8.19. Next, a large area—by means of introducing open space—with undeveloped land is claimed as illustrated in figure 8.20

A similar procedure is applied to a model that is extended with a postive external effect for the locations adjacent to the area of open space as discussed above. The results are plotted in figures 8.21 and 8.22. The values for the GE-WTA, based on (8.7.3), are plotted as the percentage of the income in figure 8.23. Similar values from a different series of runs with the same parameter values are plotted in figure 8.24.

The results from both series reveal a compensation for the higher income groups for the loss of residential space in terms of a benefit from positive external effects when the vicinity of open space is accounted for as an external effect. Especially the second series indicates the transfer of the GE-WTA from the group with the highest

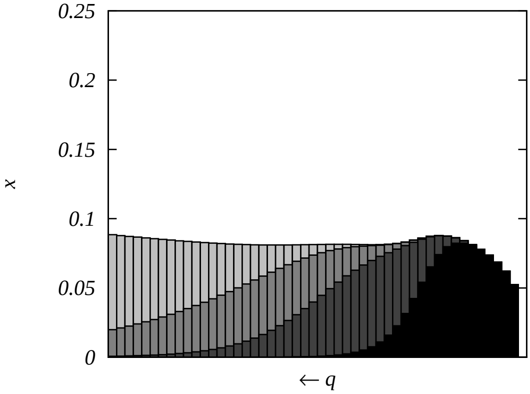

Figure 8.18: *One dimensional sorting equilibrium.*

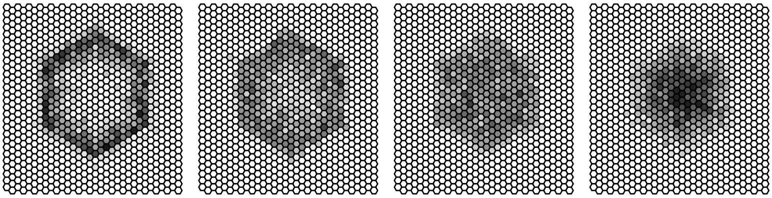

Figure 8.19: *Sorting equilibria per income group (no external effects).*

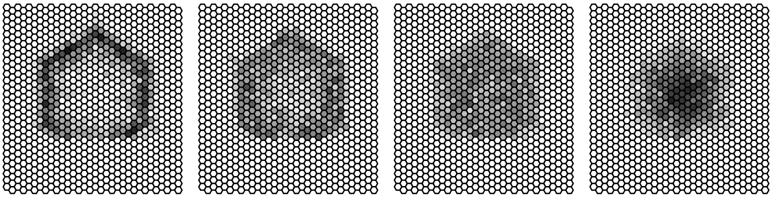

Figure 8.20: *Sorting equilibria per income group with open space (no external effects).*

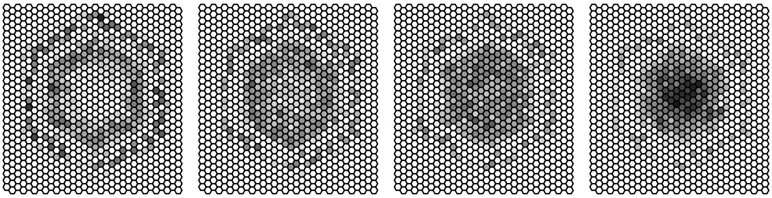

Figure 8.21: *Sorting equilibria per income group (with external effects).*

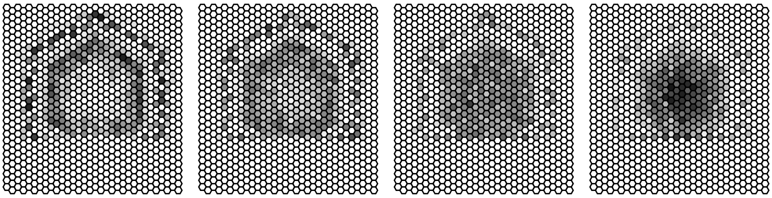

Figure 8.22: *Sorting equilibria per income group with open space (with external effects).*

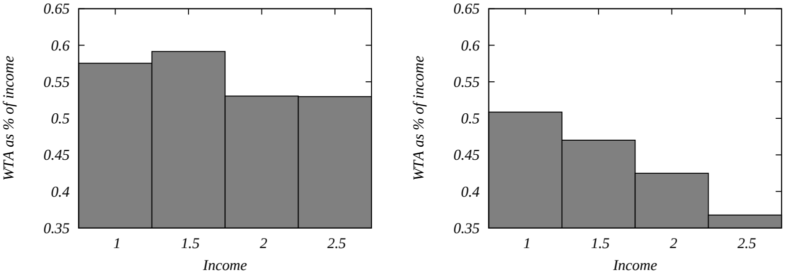

Figure 8.23: *GE-WTA as percentage of income, per income group, without and with external effects.*

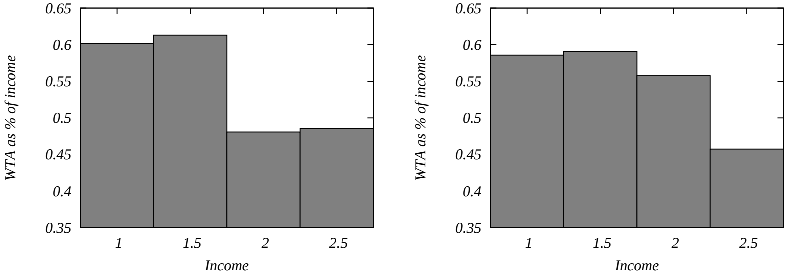

Figure 8.24: *GE-WTA as percentage of income, per income group, without and with external effects (second series).*

income to the group with the second-highest income. An explanation might be that due to the external effect of open space at the locations next to the undeveloped area, price competition primarily results in a relocation of the higher income group next to the open space. Since this group is already dominant at the CBD, a kind of corridor is established from the CBD to the undeveloped area where the high incomes are in the majority. This leaves the two groups with lower incomes nearly unaffected, as they already reside in the suburbs. Only the group with the second-highest income is seriously affected because a smaller area remains for them between the CBD and the suburbs, around the corridor for the highest incomes.

This experiment illustrates how sensitive and differentiated welfare effects from land policy can become if a welfare assessment in spatial equilibrium is undertaken and the impacts of complex external effects are accounted for. Although it is difficult to derive policy implications from this experiment that can be assumed to be generally valid, the qualitative result might be contrasted with the ‘neoclassical intuition’. Since the runs without external effects are based on the Alonso variant, the resulting land use pattern is essentially ‘optimal’ in the neoclassical sense. The GE-WTA in this experiment is a measure for the welfare loss, due to the creation of an area of open space. As the diagrams on the left in figures 8.23 and 8.24 reveal, the welfare losses are not distributed equally over the income groups. In this experiment the welfare loss for the lower income groups is relatively higher, because they resided at the locations where the area of open space was created. New locations in the remaining suburbs are shown to offer only limited compensation. This equity aspect of land use planning can only be assessed in spatial equilibrium, but appeals to logical reasoning. Even though the land use pattern is Pareto-optimal, an unequal distribution of welfare losses might require an additional compensation, if this is politically preferable.

If external effects are taken into account as endogenous amenities, optimal policy measures might become less intuitive. Although the higher income group was expected to be compensated most for the welfare loss—because of the benefits from new attractive locations next to the open space area—the second run of the experiment shows that these benefits might become even larger, at the expense of the second-highest income group. As a result, the welfare loss of the latter is larger than in the case without external effects. The presence of endogenous amenities might therefore have consequences for compensation policies that can only be discovered using computer simulations. However, due to the presence of multiple equilibria in this type of model—depending on the specification of the endogenous amenities—the possibilities for conducting quantitative analyses using econometric estimation methods might be limited.

8.8 Discussion and conclusions

This chapter introduced the new concept of a Multi-Agent Discrete Choice Model (MADCM). An MADCM can be considered a discrete choice model with interactions, implemented as a multi-agent system (MAS). To the knowledge of the author, no examples of this specific combination exist in literature. The theoretical considerations as well as a number of technical details concerning the construction of an MADCM were discussed in this chapter.

A wide range of possible externalities was explored using an MADCM implementation of the model developed in chapter 6. The presence of externalities combined with evolutionary dynamics can be interpreted as a self-organising market economy. Chapter 7 already concluded that the role of a government facing *agglomeration externalities* is likely to be different from the traditional position defined in neoclassical microeconomics, stressing the internalisation of external effects in order to achieve a Pareto-optimal allocation. Chapter 8 amplifies this observation. Chapter 7 argued that agglomeration externalities might be considered rather endogenous amenities, although internalisation was in theory possible for network externalities, where only the presence of other agents at the same location affected the level of well-being of the individual agent. Internalisation would be a nearly impossible task for the more complex externalities discussed in this chapter, because the dependency is extended to more locations.

As the Alonso model is part of the neoclassical tradition, it is not surprising that the variants presented in this chapter with only an *exogenous* local quality vector yield a unique solution. Results from neoclassical economics can then be transferred to a spatial context. Emergent agglomerations can only be explained by the presence of positive externalities. Externalities not only jeopardise the Pareto efficiency of the equilibrium solution, they can also give rise to multiple equilibria. Since agglomeration externalities are positive external effects, they can give rise to a positive feedback mechanism at the system level and be interpreted analogous to the *coordination game* in game theory, presented in chapter 3. If the attractiveness of a location is primarily dependent on the presence of neighbours, the agent might as well choose a different location together with the group of neighbours. If the agent and the neighbours do not coordinate their collective location decision, the history of the sequence of individual location decisions will determine the locations chosen by the group. From a research point of view, this type of path dependency poses a problem because the number of equilibria can become very large. The model supports only a qualitative interpretation, in the sense that if history determines the equilibrium outcome completely, the resulting land use pattern becomes nearly random. As a consequence, a quantitative interpretation—including econometric estimation—might become impossible.

However, if the set of possible equilibria can be classified in a limited number of types, each type could still be assigned a specific interpretation. Again, similar to a coordination game, there might be a ranking of equilibria possible in terms of their payoffs. A limited set of possible land use patterns can be ranked according to their level of *spatial social welfare*. In that case, all equilibria correspond to a Nash equilibrium, but only one will be *Pareto superior*. If the current land use pattern is Pareto inferior, a theoretical possibility for a public policy leading toward a *transition* to the optimal solution exists.

In addition to the possible existence of a lock-in situation for the current land use pattern, additional caution must be taken with the welfare interpretation of the current pattern as a result of imperfect information. Simulation runs in which the number of candidate locations is limited for the agent who improves her location choice, reveal the possibility of transient agglomerations that might eventually be replaced by a different land use pattern, yielding a higher level of social welfare. In the context of land policy, these disequilibrium dynamics suggest that an existing configuration might be unstable.

The final experiment in this chapter focuses more directly on the welfare effects of capitalisation in relation to resorting. Section 8.7 showed how the effect of external effects from the vicinity of open space—in addition to the value of open space as a pure public good as discussed in chapter 7—can give rise to equity concerns. Due to the capitalisation of the value of open space as an externality, it is possible that only higher income groups can afford living next to the open space. In the experiment conducted in this chapter, following the relocation after the creation of an 'eco zone' resulted in a significant transfer of welfare loss from the group with the highest income to the group with the second highest income. Although this result is difficult to generalise, it illustrates the sensitivity of the equity considerations on how endogenous amenities are defined.

Chapter 9

Summary and conclusions

9.1 Introduction

This thesis explores several aspects of a welfare economic approach to land use and land use planning. The main aspects addressed are the optimal allocation of land, the capitalisation of amenity values in land prices and the value of open space. The decision was taken to address these welfare issues while developing a modelling framework. The final model is suitable for presenting a unifying approach to the aspects mentioned above. Although the final model is bound by the use of a certain type of functional form—a utility function with a constant elasticity of substitution (CES)—its specification can accommodate a wide range of cases. Furthermore, the chosen specification allows for a consistent integration of traditional economics and an agent-based computational approach. It offers an evolutionary perspective on the land market, while simultaneously maintaining a relation with neoclassical welfare economics.

In this final chapter, section 9.2 presents a summary of the previous chapters. Section 9.3 draws conclusions based on the answers to the research questions of chapter 1.

9.2 Summary

Beginning with a sketch of the general position of a government relative to a market in the neoclassical framework of microeconomics in chapter 1, a few issues were raised. First, a spatially explicit assessment has only recently become part of mainstream economics mainly due to the impossibility of accounting for the emergence of agglomerations in the general equilibrium model by Arrow and Debreu (1954), that is dominant in economic theory and by consequence also in public sector economics. In economics, the *emergence of an agglomeration* refers to the formation of a cluster of consumers, producers, or both, because of economic reasons. With the neoclassical framework it can be shown that under specified conditions the allocation of goods by a market is optimal. These conditions have two implications for the behavioural model at the level of individual agents. The first implication is the elimination of all interactions other than those mediated by prices. If only market interactions are accounted for, agents can not choose to locate in the vicinity of each other even when that would facilitate social interactions. In other words, social communication can not be integrated in the neoclassical framework. Producers cannot benefit from the spillover effects on their productivity that might result from the presence of other firms in the same area, followed by informal exchanges of ideas. In the neoclassical framework, social interactions and spillovers are identified as *external effects*, jeopardising the efficiency of the market allocation. The second implication results from the condition that production takes place only at non-increasing returns to scale. Increasing returns are equivalent to product differentiation. If there is a preference for variety on the consumption side of an economy, two products from different brands for example, are not perfect substitutes for the consumer. As a consequence, consumers are willing to pay slightly more for their preferred variety and the producers need not sell their product for a market price equal to the marginal cost of production. Hence, their profit will increase with the volume of their production. As product differentiation is effectively not allowed in the neoclassical framework, products can not be distinguished by their *location* of production. As a result, clusters of production cannot be accounted for in neoclassical economics.

As both non-market interactions and product differentiation are considered market distortions in the neoclassical framework, the traditional normative role of a government would consist of eliminating them, to restore the optimal market allocation. However, because agglomerations can only be explained by assuming the presence of these distortions, the welfare economic position of a government in a spatially explicit economic context needs to be redefined. Regarding land policy, this position specifically concerns the economics of land use with respect to agglomerations. The starting point for redefining the role of a government chosen in this thesis is the relatively recent adoption of models in mainstream economics that serve as alternatives to the neoclassical framework. Many have an origin in the research on *complex systems*.

Since a complexity approach to land use theory is occasionally presented as a radical alternative to neoclassical economics, its origin is discussed in more detail in chapter 2. There, it is argued how mathematical biology—and especially *evolutionary game theory*—offers a methodological basis for integrating a *systems approach* with *methodological individualism*. The first is needed to accommodate insights from the theory on complex dynamical systems in a model, especially regarding adaptation and self-organisation. The latter supports the justification of adopting a proxy for fundamental laws of human behaviour, if at the individual level the behaviour is specified as conditional rule-based decisions. Individual-based and systems approaches are therefore not necessarily opposites. In an evolutionary perspective, both can be considered complements. Individual decision rules define the postulates of a deductive system. Their position in the model can be stressed, if in a computational model the algorithm is implemented as individual objects equipped with these rules, resulting in an agent-based model. This approach is sometimes called a *generative* approach to social science (Epstein, 2006).

Chapter 3 focuses the discussion of chapter 2 more specifically on the topic of complexity. Although complexity science is sometimes presented as a field of research itself, it does not offer a ‘complexity theory’. Chapter 3, therefore, identifies two types of complexity relevant for answering the research questions in this thesis. The first is related to *complex dynamical systems*, usually concerning coupled subsystems with non-linear feedback mechanisms. As the general descriptions of these systems adhere to a systems approach, in line with chapter 2, the role of complexity in evolutionary game theory is stressed in order to arrive at an interpretation at the level of individual behaviour. Especially the position of the so-called *coordination game* is highlighted, as it supplies a consistent interpretation of multiple equilibria, bifurcations, path dependency, and lock-in. The second type of complexity concerns *computational complexity*. Computational complexity deals with the issue of the possibility of and time involved in solving mathematical problems. It does so by giving a definition of computation that relies on the description of a hypothetical machine for implementing recursive functions: the Turing Machine. Because the design of modern computers is still based on this description, the definition of computation can be used in the argumentation that computer models, including agent-based models, are not fundamentally different from models consisting of equations. This observation is consistent with the use of agent-based models in the generative, deductive approach discussed in chapter 2. These two types of complexity do not immediately refer, however, to a complementarity with neoclassical economics. Therefore, in chapter 3 both are related to the role linear algebra plays in the neoclassical framework. Following Koopmans (1957), the conditions—or postulates—that define the behaviour of individuals in neoclassical economics ensure that the choice sets of consumption and production are linearly independent. As a consequence, what is optimal for the individual agent is also best for society, defined as the sum of all individuals. The postulates are identical to the conditions referred to in chapter 1, limiting the interactions between agents to market interactions only. The role of complexity in neoclassical economics can therefore be identified with the problem of aggregation and rationality in the presence of *non-market interactions*.

Chapter 4 applies the material collected in chapter 2 and 3 directly to the elementary context of a two-agent two-goods pure exchange economy. Following Bowles (2004) the Pareto efficiency of market allocation—that is at the heart of neoclassical economics and defining the role for the state—is considered to be problematic. Although its normative interpretation is clear, the formation of equilibrium prices at which supply is equal to demand lacks a consistent interpretation as a process. In chapter 2 evolutionary game theory was already selected as a framework that allows for an integration of a systems perspective on self-organisation with individual rule-based behaviour. This framework is adapted in chapter 4 to accommodate an evolutionary approach to market clearing. The evolutionary approach developed is based on best response dynamics, in a way that—to the knowledge of the author—is a new addition to the existing literature. Based on a utility function with a constant elasticity of substitution (CES) and its relation with the logit model in the literature on discrete choice (Anderson et al., 1992), a simple rule-based decision is defined that corresponds to a best response to a given price. This best response can be interpreted as the fraction of his income the agent prefers to spend on the good. The demand function can be reconstructed from this by dividing the fraction of the income by the price. This approach allows for a translation of the maximisation of a utility function, to the direct use of the corresponding indirect utility function in a simple behavioural rule. Two implementations of this model are explored. In accordance with the neoclassical metaphorical interpretation, an auctioneer is introduced who quotes disequilibrium prices. The prices are shown to converge to the same equilibrium prices as in the neoclassical model. For relatively high values of the CES however, the process may fail to converge and the chaotic price dynamics that follows are caught in a strange attractor. In the second implementation, the role of the auctioneer becomes superfluous and the agents are involved in a bargaining process. This process converges to the neoclassical equilibrium solution as well; also for relatively high values of the CES. The bargaining model presented in chapter 4 is different from the existing literature on bargaining games, and is actually inspired by the literature on learning in games. Its implementation conforms to a minimum level of cognitive capabilities and thereby to a generative approach to the neoclassical market equilibrium. Chapter 4 also introduces rudimentary extensions to product differentiations and the interpretation of network externalities analogous to the coordination game in chapter 3.

Chapter 5 presents an overview of the existing economics literature on land use and welfare, highlighting similarities with the basic concepts applied in the context of a two-agent bargaining process in chapter 4. From a welfare perspective, the similarities between the literature on the capitalisation of the value of amenities in the market price for land in urban economics, public finance, and environmental economics are stressed. Concerning complexity issues, the emergence of agglomerations, and game theoretic assessments of interactions, the role of regional economics is considered leading after the introduction of the *New Economic Geography* (Krugman, 1991; Fujita et al., 1999). Although the models show affinity at a conceptual level with the issues discussed in the previous chapters, land use is usually not addressed in regional economics. Special attention is devoted in chapter 5 to location choice models recently introduced in environmental economics. Originally intended for extending the hedonic pricing method, in principle they provide a basis for integrating welfare concepts from urban economics and environmental economics. The first concerns optimal land use, the latter concerns the capitalisation of amenity values. Furthermore, the econometric estimation procedures suggested for these so-called locational sorting models, are based on the literature on discrete choice with social interactions. This literature offers a complexity perspective on the relation between discrete choice models and statistical mechanics and shows similarities with evolutionary game theory.

Chapter 6 develops an evolutionary reinterpretation of the Alonso model in urban economics. An extension of this model with stochastic error terms supports two interpretations. Based on the analogy with discrete choice models, the model can be interpreted as a population game. Interpreted as a model of a representative consumer, a direct relation with the two-agent model developed in chapter 4 is evident. The latter interpretation facilitates the welfare analysis, conducted in chapter 7. The first interpretation is the basis for the implementation as a multi-agent system (MAS) in chapter 8. Both can be related to a utilitarian social welfare function that reflects the level of welfare in spatial equilibrium. In the population game interpretation it is consistent with the evolutionary notion of fitness; for the representative consumer it is simply the indirect utility function.

Chapter 7 is devoted to several types of welfare analysis. First, the relation between locational sorting models and hedonic pricing is discussed. Locational sorting models are originally developed as an extension of hedonic pricing models. Whereas hedonic pricing is primarily suited for estimating the willingness to pay (WTP) for a *marginal* change in local amenity level, locational sorting defines a WTP on the basis of endogenous land prices, that result from a new market equilibrium in response to *non-marginal* changes in amenity levels. Locational sorting is especially interesting in the context of this thesis, because it offers a consistent interpretation of the welfare effects of capitalisation. Section 7.3 reintroduced the network externalities from chapter 4 in the model of chapter 6 as local agglomeration externalities. The main result of the welfare analysis, is that, although internalisation of the value of these externalities—restoring the Pareto efficient allocation of land—is possible in theory, an interpretation as *endogenous amenities* is more appealing. Finally, chapter 7 shows that the model from chapter 6 can in principle also be applied to assess the value of open space. In the existing locational sorting models the total supply of land for residential use is usually assumed to be fixed. Since the level of social welfare in spatial equilibrium in the model of chapter 6 also depends on the number of locations that are developed, the value of open space can be related to a *general equilibrium willingness to accept* (GE-WTA) in a society for not enjoying more residential space. Furthermore, it is proven that with a translation of this GE-WTA to equivalent opportunity costs, the concerning parcel would not be developed. This interpretation offers a theoretical basis for uniting the virtual value of land from valuation methodology with the real market price for the same area.

The model developed in chapter 6 is translated to a special type of a multi-agent system (MAS) in chapter 8. This chapter introduces it as a *Multi-Agent Discrete Choice Model* (MADCM). There are economic theoretical, as well as computational theoretical, motivations for this translation. In an MADCM, individual agents are equipped with a decision rule and assigned the values of draws from the distribution of the idiosyncratic component in the preference structure of the equivalent logit choice model. This procedure resembles Monte Carlo integration methods in econometrics. It allows for regarding a simulation result as one of the many possible realisations, instead of the more abstract interpretation in terms of probabilities in chapter 6. Additionally, it facilitates a consistent interpretation of the results with a finite number of agents. The chapter illustrates the possibility of introducing more complex types of externalities and how they can be accommodated in the evolutionary development within the model. While complex externalities can result in patterns that might seem more realistic at first glance, the number of possible equilibria also increases. Therefore only rather stylised situations can still be accounted for, though these situations do offer insights into the phenomena that can be important in the context of land policy. After it is shown that the MADCM can accommodate several classical examples of land use models, a number of experiments is conducted. The first concerns the possibility that current land use patterns reflect a lock-in situation, instead of an optimal configuration. With a combination of evolutionary development, an endogenous agglomeration—based on social interactions—and the presence of obstacles in a grid, multiple equilibria can be classified in distinct types. An example is presented with two types, where one is Pareto-superior to the other. In the second experiment, the number of locations the individual agent can select for improving his level of well-being is restricted. This can be considered as an example of imperfect information. Simulation runs for certain parameter values show evolution toward an optimal land use pattern that may contain one or several transient configurations. This experiment reveals a different reason for caution with the identification of the current land use pattern as optimal. The third experiment concerns an extension of the assessment of open space in chapter 7. If open space is not only considered a pure public good, but also the source of positive externalities, equity concerns may be different. In a model where the group with the highest income is located closest to an exogenously given CBD, this same group is also likely to be able to afford higher market prices for locations next to an area with open space, if open space is accounted for as an externality and its value is capitalised in the market price for land. In the simulation run presented, the second highest income group is driven away from their original locations as a result of capitalisation, thereby facing a higher welfare loss when open space externalities are taken into account.

9.3 Conclusions

The research questions formulated in chapter 1 can now be answered. They are the basis for the conclusions of this thesis.

9.3.1 Welfare economics for a land market

The first question concerns the translation of concepts from traditional welfare economics to the context of land policy. This translation was expected to be problematic, especially because of the need for agglomerations externalities if emergent agglomerations have to be accounted for. Since a market allocation with external effects is not Pareto efficient in the neoclassical framework of microeconomics, this question was initially intended to address the efficient allocation of land in the presence of agglomeration externalities. However, as was noted already in chapter 1, the allocation of land usually also involves the allocation of characteristics of different parcels. If the discussion is restricted to residential land use, the location characteristics—in a first assessment—might be considered local pure public goods, or amenities. This means that at the level of a location, resident consumers benefit from the presence of the amenities but cannot be excluded from consumption individually. This does not mean however, that the individuals do not pay for the amenities, as their value in part determines the market price for land at the location. If land is considered a differentiated good with the amenity on it corresponding to its quality level, the capitalisation of the value of a local pure public good does not affect the efficiency of the market allocation of the land.

This perspective offers an alternative interpretation of agglomeration externalities, as they can also be considered *endogenous* amenities. Although this perspective does affect the efficiency of land in the strict neoclassical sense, it is consistent with the usual interpretation of a pure public good as an extreme case of an externality. The price of land might therefore not support an efficient allocation of the land itself, but the price partially capitalises the value of the externalities. The role of government in terms of spatial welfare economics is therefore likely to be concerned with the level of social welfare in spatial equilibrium, instead of efficiency considerations. For the individual agent this level of spatial welfare is identical to the expected value of the indirect utility function for all locations. The spatial equilibrium is defined as the market equilibrium on the land market, with the locations as varieties of land as a differentiated good.

9.3.2 The role of complexity

The second research question concerns the role of theories on complex systems with respect to price formation on a land market. Similar to the first research question, this question was originally supposed to deal with the presence of agglomeration externalities. With some applications of complexity science already present in regional economics and social economics, the use of computer simulations was likely to be necessary, because as with the interpretation of externalities as non-market externalities, typical ‘complexity phenomena’ such as the existence of multiple equilibria were expected.

To answer this question a different strategy was finally chosen. The neoclassical framework assumes that markets clear, although the framework does not offer a description of the process in which equilibrium prices are established. This omission is of special interest in the context of policy in which market efficiency is implicitly presented as the result of an emergent process. Although a substantial part of the existing literature on social interactions originates in—or at least shows similarities with—evolutionary game theory (EGT), no direct applications of EGT were found that deal with a reinterpretation of the standard examples of neoclassical economics. Before assessing the role of non-market interactions in this thesis, market interactions were cast in a framework of best response dynamics to allow for an interpretation of an efficient market allocation in terms of self-organisation.

This approach was limited to utility functions with constant elasticity of substitution (CES), but nevertheless offers—under certain conditions—a way of interpreting a market economy as a self-organising system. A second benefit of adopting a best response approach to a CES utility function is the possibility of a direct interpretation of the indirect utility function in terms of a relatively simple decision rule. Furthermore, this interpretation was consistent in the context of two agents as well as in the context of populations. And since in the answer to the first research question it was argued that the level of well-being in spatial equilibrium essentially corresponds to the indirect utility function, this rule-based interpretation is still consistent with an assessment of spatial welfare.

Because external effects can be accommodated in this framework as relatively simple extensions, the answer to the second research question needs to be differentiated. Evolutionary game theory in conjunction with CES utility functions enables an interpretation of an economy as a self-organising system. In the presence of external effects, this resulting equilibrium might not be unique and the possibility of path dependency and lock-in situations need to be taken into account. Finally, although the value of some types of externalities can in principle be internalised in the price of land—in order to restore the Pareto-efficiency of the allocation—, an interpretation as endogenous amenities is preferable. This interpretation was part of the answer to the first research question. Without internalisation, external effects can still easily be integrated in the rule-based decision; supporting a more evolutionary perspective on the market as self-organising system.

9.3.3 Policy implications

Regarding the third question, concerning the applicability of spatial welfare economic concepts in land policy and land use planning, mainly conceptual insights are offered. With respect to the position of a government relative to a land market, the main conclusion following the answers to the first two research questions, is that a strict neoclassical perspective on the allocation of land as a market good does not necessarily maximise social welfare. Internalisation of the value of external effects is in most cases not possible. It is also more consistent from a theoretical point of view to consider many external effects as *endogenous amenities*.

Central to the answer to the first research question is the concept of *spatial welfare*. The operationalisation of the concept is relatively difficult. Following the existing locational sorting models applied in environmental economics, the measurement of a willingness to pay for improvements in amenity levels that is corrected for endogenous land or housing prices is possible under some restrictions. However, these methods usually assume a fixed total supply of land. In this thesis it was demonstrated that for a theoretical model this supply can be endogenised. Further research is needed on the possibility of integrating an endogenous supply of land in econometric estimation. However, a trade-off always needs to exist between the degree of realism possible and the derivation of quantitative results due to the existence of multiple equilibria in models with more complex externalities.

The concept of spatial welfare also plays a role in the assessment of the value of open space. Even without the operationalisation and measurement of a real value, the approach sketched in chapter 7 offers a conceptual perspective on the preservation of open space that is under the pressure of expanding cities. Instead of measuring a willingness to pay for a protected area, the value of undeveloped land might be assessed in terms of a general equilibrium willingness to accept the forgone conclusion of a city’s expansion. This GE-WTA would have to be determined by using a spatial social welfare function; but above all a consistent interpretation requires that society defines opportunity costs that reflect contribution to the level of welfare from virtual income from undeveloped land. The result from the MAS experiment in which the part of the value of open space is also capitalised in the value of the adjacent locations, stresses the need for a differentiation of benefits according to individual characteristics, especially income. It means that if open space is concerned society is required primarily to consistently valuate what it has, rather than to pay for something a government could acquire.

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Summary in Dutch

In theorieën rond de economie van de publieke sector is de taakverdeling tussen overheid en markt veelal gebaseerd op het uitgangspunt dat goederen in beginsel optimaal door een markt kunnen worden gealloceerd. De overheid dient vooral in te grijpen in speciale gevallen die worden opgevat als *verstoringen van de markt*. De centrale vraag in dit proefschrift is hoe deze taakverdeling gedefinieerd dient te worden op het gebied van *ruimtelijke ordening*. Is de allocatie van ruimte door een grondmarkt ook optimaal? Of kan de overheid alleen aan haar maatschappelijke taak voldoen door ruimtegebruik te plannen? Hoe draagt ruimtelijk ordeningsbeleid bij aan maatschappelijke welvaart? Wat zijn de voorwaarden voor een transitie naar *duurzaam ruimtegebruik*?

In dit proefschrift wordt een aantal aspecten van een welvaartseconomische benadering van grondgebruik en ruimtelijke ordening verkend. De voornaamste aspecten zijn de optimale allocatie van grond, de kapitalisering van de waarde van voorzieningen in de grondprijs en de waarde van open ruimte. Er is voor gekozen deze welvaartsonderwerpen te verenigen in de ontwikkeling van een model. Hoewel het uiteindelijke model beperkt is tot het gebruik van een specifieke functie, een nutsfunctie met een constante elasticiteit van substitutie (CES), kan met deze specificatie een groot aantal onderwerpen worden behandeld. Bovendien kan het model zodanig worden gespecificeerd dat op een consistente wijze traditionele economie met *agent-based modellering* (*agent-based modelling*) kan worden geïntegreerd. Hiermee kan systematisch de bijdrage van theorieën rond complexe systemen aan de ruimtelijk economische theorie in kaart worden gebracht. Dit proefschrift levert een bijdrage aan de bestaande literatuur door op een consistente en nieuwe wijze ogenschijnlijk zeer diverse concepten—traditionele en alternatieve—te ordenen, te combineren en toe te passen.

In hoofdstuk 1 worden de onderzoeksvragen geformuleerd. Eerst wordt de achtergrond geschetst, te beginnen met een bespreking van de positie van een overheid ten opzichte van een markt, volgens het neoklassieke kader in de micro-economie. Een ruimtelijk expliciete benadering van economische vragen maakt pas sinds zeer recent deel uit van de reguliere economische theorie. Dit is vooral het gevolg van de onmogelijkheid de *emergentie* van agglomeraties te verklaren in het algemeen evenwichtsmodel van Arrow en Debreu (1954), dat de economische theorie domineert. Het bepaalt ook voor een groot deel de manier waarop beleidsinstrumenten in de economie van de publieke sector zijn gedefinieerd. Onder de emergentie van een agglomeratie wordt in de economie de formatie van een cluster om economische redenen verstaan. Dit kan een cluster zijn van consumenten, producenten, of beide. Met het neoklassieke kader kan worden aangetoond dat onder bepaalde voorwaarden de allocatie van goederen door een markt optimaal is. Deze voorwaarden hebben twee implicaties voor het gedragsmodel op het niveau van individuele agenten. De eerste implicatie behelst de eliminatie van alle vormen van interacties, anders dan de interacties die verlopen via prijzen, ofwel *marktinteracties*. Indien in een analyse alleen marktinteracties worden meegenomen, bestaat er geen economisch gemotiveerde reden voor agenten om zich in elkaars nabijheid te vestigen, ook al zou het *sociale interacties* vergemakkelijken. Anders gezegd, sociale communicatie kan niet worden geïntegreerd in het neoklassieke kader. Ook kunnen producenten geen voordeel ondervinden van *spillovers* door de aanwezigheid van andere bedrijven in hetzelfde gebied, bijvoorbeeld door een informele uitwisseling van ideeën. Deze uitwisseling zou immers bestaan uit *niet-marktinteracties* en daarvoor is geen ruimte in het neoklassieke kader. In het kader zouden zowel sociale interacties als spillovers vallen onder de *externe effecten* die de efficiëntie van de marktallocatie in gevaar zouden brengen. De tweede implicatie volgt uit de voorwaarde dat productie plaatsvindt onder niet-toenemende meeropbrengsten. Toenemende meeropbrengsten kunnen worden beschouwd als het equivalent van *productdifferentiatie*. Wanneer er aan de consumptiezijde van de economie een voorkeur voor verscheidenheid bestaat, zullen bijvoorbeeld twee producten van verschillende merken niet volledig inwisselbaar zijn voor de consument. Als gevolg daarvan zijn consumenten bereid iets meer te betalen voor de productsoort van hun voorkeur. Dit stelt producenten in de gelegenheid hun producten tegen prijzen op de markt te brengen die boven de marginale kosten van productie liggen. Daardoor zal hun winst stijgen bij een toenemend geproduceerd volume. Omdat productdifferentiatie niet is toegestaan in het neoklassieke kader, kunnen producten niet worden onderscheiden. In een ruimtelijke context betekent dit dat producten ook niet kunnen worden onderscheiden naar locatie. Het ontbreken van toenemende meeropbrengsten in het neoklassieke kader vormt daardoor een tweede belemmering voor het verklaren van agglomeraties.

Aangezien in het neoklassieke kader zowel niet-marktinteracties als productdifferentiatie worden opgevat als verstoringen van de markt, zou de traditionele, normatieve rol van een overheid bestaan uit het elimineren van deze verstoringen, om zo de optimale marktallocatie te herstellen. Omdat agglomeraties echter alleen kunnen worden verklaard onder de aanname dat deze verstoringen aanwezig zijn, dient de welvaartseconomische positie van een overheid in een ruimtelijke context opnieuw te worden gedefinieerd. Met betrekking tot grondbeleid gaat het bij deze positie vooral om de economie van het grondgebruik in relatie tot agglomeraties. Als startpunt voor de herdefinitie van de overheidsrol is in dit proefschrift gekozen voor het relatief recente gebruik van modellen in de reguliere economie die alternatieven vormen voor het neoklassieke kader. Veel van deze modellen vinden hun oorsprong in het onderzoek naar *complexe systemen*.

Bovenstaande overwegingen leiden tot drie onderzoeksvragen:

1. Hoe kunnen traditionele concepten uit de economie van de publieke sector, zoals efficiëntie, optimaliteit en rechtvaardigheid, worden vertaald naar een grondmarkt?

2. Wat is de rol van theorieën rond complexe systemen in de formatie van grondprijzen?

3. Hoe kunnen de ruimtelijke equivalenten van welvaartseconomische concepten worden toegepast in beleid?

In hoofdstuk 2 wordt gesteld dat mathematische biologie—en met name *evolutionaire speltheorie*—een methodologische basis biedt voor het integreren van een systeembenadering en methodologisch individualisme. Een systeembenadering is vereist voor het onderbrengen van inzichten uit de theorie van complexe dynamische systemen, vooral met betrekking tot adaptatie en *zelforganisatie*, in een model. Anders dan de natuurwetenschappen kennen de sociale wetenschappen geen universele wetten. Deze kunnen eventueel wel worden benaderd, indien gedrag op het niveau van het individu wordt gespecificeerd als voorwaardelijke, op regels gebaseerde beslissingen. In een evolutionair perspectief geven bepaalde regels en beslissingen op het niveau van individuen aanleiding tot complex gedrag op geaggregeerd niveau. Individuele beslisregels kunnen worden opgevat als de definities van postulaten in een deductief systeem. Met een model worden in dat geval niet de implicaties van universele wetten verkend, maar van de aannamen die de modelleur maakt. De positie van individuele beslisregels in een model kan worden benadrukt wanneer in een numeriek model het algoritme wordt geïmplementeerd als een verzameling afzonderlijke objecten die zijn uitgerust met deze regels. Een dergelijk model wordt een *agent-based model* (ABM) genoemd. Een ABM kan een verklaring geven voor een sociaal-wetenschappelijk fenomeen mits het fenomeen met een simulatie kan worden gereproduceerd. Deze aanpak wordt soms een *generatieve* benadering van de sociale wetenschappen genoemd (Epstein, 2006).

In hoofdstuk 3 wordt de discussie meer specifiek gericht op het onderwerp ‘complexiteit’. Hoewel ‘complexiteitswetenschap’ soms wordt gepresenteerd als een zelfstandig onderzoeksgebied, bestaat er geen eenduidige ‘complexiteitstheorie’. Daarom worden in hoofdstuk 3 twee vormen van complexiteit geïdentificeerd die van belang zijn voor het beantwoorden van de vragen in dit proefschrift. De eerste vorm is gerelateerd aan complexe dynamische systemen en heeft meestal betrekking op aan elkaar gekoppelde deelsystemen met niet-lineaire terugkoppelmechanismen (*feedback*). Omdat de algemene beschrijving van dergelijke systemen overeenkomt met de systeembenadering zoals besproken in hoofdstuk 2, wordt in hoofdstuk 3 de rol van complexiteit in evolutionaire speltheorie benadrukt om te komen tot een interpretatie op het niveau van individueel gedrag. Vooral de positie van het zogenaamde *coördinatiespel* wordt belicht, omdat het een consistente interpretatie biedt voor meervoudige evenwichten, bifurcaties, padafhankelijkheid en een lock-in. De tweede vorm van complexiteit die van belang is in de volgende hoofdstukken is *numerieke complexiteit* (*computational complexity*). Numerieke complexiteit heeft betrekking op de mogelijkheid om wiskundige problemen op te lossen en de tijd die dat kost. Het hanteert een definitie van berekening die berust op een beschrijving van een denkbeeldige machine voor het implementeren van recursieve functies, de *Turing Machine*. Omdat het ontwerp van moderne computers nog altijd is gebaseerd op deze beschrijving, kan de definitie van berekening worden gebruikt ter onderbouwing van het betoog dat computermodellen, waaronder agent-based modellen, in fundamenteel opzicht niet verschillen van modellen die bestaan uit vergelijkingen. Dit betoog is consistent met het gebruik van agent-based modellen in de generatieve, deductieve benadering zoals besproken in hoofdstuk 2. Deze twee vormen van complexiteit verwijzen niet onmiddellijk naar een mogelijke relatie met de neoklassieke economie. Daarom worden beide aan het einde van hoofdstuk 3 eerst in verband gebracht met de rol die lineaire algebra speelt in het neoklassieke economische model. Koopmans (1957) toont aan dat de voorwaarden—of postulaten—die het menselijk gedrag definiëren voor het neoklassieke kader afdwingen dat de keuzeverzamelingen voor consumptie en productie *lineair onafhankelijk* zijn. Als gevolg daarvan is wat optimaal is voor het individu, ook het beste voor de maatschappij. Dit volgt omdat de maatschappij is gedefinieerd als de *som* van alle individuen. Deze postulaten zijn dezelfde als waarnaar verwezen werd in hoofdstuk 1, waar werd benadrukt dat zij de interactie tussen agenten beperken tot alleen marktinteracties. De rol van complexiteit in de neoklassieke economie kan daarom worden geïdentificeerd met het probleem van aggregatie in de aanwezigheid van niet-marktinteracties.

In hoofdstuk 4 wordt het in hoofdstuk 2 en 3 verzamelde materiaal toegepast op de elementaire context van een pure ruileconomie met twee agenten en twee goederen. In navolging van Bowles (2004) wordt de Pareto-efficiëntie van de marktallocatie—die de kern vormt van de neoklassieke economie en tevens de rol van de overheid definieert—als problematisch beschouwd. Hoewel de normatieve interpretatie duidelijk is, bestaat er geen eenduidige, consistente interpretatie voor de formatie van evenwichtsprijzen—waarvoor aanbod gelijk is aan vraag—als *proces*. In hoofdstuk 2 was evolutionaire speltheorie al gekozen als een kader dat geschikt is voor de integratie van een systeemperspectief op zelforganisatie en individuele beslissingen gebaseerd op regels. Dit kader wordt in hoofdstuk 4 zodanig aangepast dat het geschikt is voor een evolutionaire selectie van een marktevenwicht. Het selectiemechanisme is gebaseerd op *beste repliek-dynamica* (*best response dynamics*), op een wijze die—voor zover bekend bij de auteur—een nieuwe toevoeging betekent aan de bestaande literatuur op dit gebied. Gebaseerd op een nutsfunctie met een constante elasticiteit van substitutie (CES) en haar relatie met het logit-model in de literatuur rond *discrete keuze* (*discrete choice*) (Anderson et al., 1992), wordt een eenvoudige, op regels gebaseerde keuze gedefinieerd die overeenkomt met een beste repliek op een gegeven prijs. De strategische keuze betreft de fractie van het inkomen die de agent wenst te besteden aan een bepaald goed. Deze strategie kan worden vertaald naar een vraagfunctie, door de fractie te delen door de prijs. In plaats van het maximaliseren van een directe nutsfunctie, kan de bijbehorende indirecte nutsfunctie worden afgeleid als verwachtingswaarde op basis van bovenstaande strategieën. Twee implementaties van dit model worden verkend. In overeenstemming met de neoklassieke, metaforische interpretatie van de totstandkoming van een marktevenwicht wordt een veilingmeester geïntroduceerd. Anders dan in het neoklassieke model, maakt de veilingmeester telkens de prijzen ‘uit evenwicht’ (*disequilibrium*) bekend. Er wordt aangetoond dat in een herhaald proces van het bepalen van de beste repliek door beide agenten, de prijzen ‘uit evenwicht’ naar de evenwichtsprijzen van neoklassieke model convergeren. Bij relatief hoge waarden voor de CES kan het echter voorkomen dat prijs niet convergeert naar een vast punt, waarna de daarop volgende chaotische prijsdynamica eventueel in een ‘vreemde attractor’ terecht kan komen. In de tweede implementatie wordt de rol van de veilingmeester overbodig gemaakt, door de agenten te verwikkelen in een onderhandelingsproces. In dit proces convergeren de prijzen naar het neoklassieke evenwicht, ook bij relatief hoge waarden voor de CES. Het onderhandelingsmodel dat gepresenteerd wordt in hoofdstuk 4 verschilt van de bestaande literatuur inzake onderhandelingsspellen; het is vooral geïnspireerd door de literatuur over *leren in spellen*. De implementatie komt overeen met een minimum aan cognitieve vaardigheden en voldoet daarmee aan de eisen voor een generatieve benadering van het neoklassieke marktevenwicht. In hoofdstuk 4 worden ook eenvoudige uitbreidingen naar productdifferentiatie geïntroduceerd, alsmede een interpretatie van netwerkexternaliteiten in analogie met het coördinatiespel uit hoofdstuk 3.

In hoofdstuk 5 wordt een overzicht gepresenteerd van de bestaande literatuur op het gebied van grondgebruik en welvaart. Hierin worden de overeenkomsten benadrukt met de concepten die in hoofdstuk 4 zijn toegepast in de context van een onderhandelingsproces met twee agenten. Wat betreft de behandeling van het onderwerp ‘complexiteit’, de emergentie van agglomeraties en de speltheoretische benadering van interacties, wordt de rol van regionale economie—met de introductie van de Nieuwe Economische Geografie (Krugman, 1991; Fujita et al., 1999)—als leidinggevend beschouwd. Hoewel de modellen van de NEG op conceptueel niveau sterke overkomsten vertonen met de onderwerpen die zijn besproken in de voorgaande hoofdstukken, wordt grondgebruik in de regionale economie meestal niet behandeld. Met betrekking tot het onderwerp ‘welvaart’ worden de overeenkomsten in de literatuur over de kapitalisering van de waarde van de voorzieningen in de marktprijs voor grond in urbane economie (*urban economics*), openbare financiën en milieueconomie beklemtoond. Voorts wordt in hoofdstuk 5 speciale aandacht gegeven aan de locatiekeuzemodellen die recent zijn geïntroduceerd in de milieu-economie. Oorspronkelijk bedoeld als uitbreiding van de *hedonische prijsmethode* (*hedonic pricing method*), reiken zij in beginsel een basis aan voor de integratie van welvaartsconcepten uit urbane en milieu-economie. De eerste hebben betrekking op optimaal grondgebruik, de tweede op de kapitalisering van de waarde van voorzieningen. Bovendien zijn de econometrische schattingsprocedures die worden voorgesteld voor deze zogenaamde ‘locationele sorteermodellen’ (*locational sorting models*) gebaseerd op de literatuur over discrete keuze met sociale interacties. Deze literatuur biedt een complexiteitsperspectief op de relatie tussen discrete keuzemodellen en statistische mechanica. Bovendien bestaat er een verwantschap met evolutionaire speltheorie.

In hoofdstuk 6 wordt een evolutionaire benadering van het Alonso-model uit de urbane economie ontwikkeld. Een uitgebreide variant van dit model, waarin een stochastische foutterm is toegevoegd, blijkt twee interpretaties te ondersteunen. Gebaseerd op de analogie met discrete keuzemodellen, kan het model worden opgevat als een populatiespel. In de interpretatie als model van een representatieve consument bestaat er een directe relatie met het twee-agentenmodel zoals ontwikkeld in hoofdstuk 4. De laatste interpretatie vergemakkelijkt de welvaartsanalyse die wordt uitgevoerd in hoofdstuk 7. De eerste interpretatie vormt de basis voor de implementatie als een *multi-agentsysteem* (*multi-agent system*; MAS) in hoofdstuk 8. Beide interpretaties kunnen worden gerelateerd aan een utilitaire sociale welvaartsfunctie die het *welvaartsniveau in ruimtelijk evenwicht* weergeeft. In de interpretatie als populatiespel is deze functie consistent met het evolutionaire begrip ‘geschiktheid’ (*fitness*), terwijl het voor de representatieve consument eenvoudig de indirecte nutsfunctie is.

Hoofdstuk 7 is gewijd aan verschillende typen welvaartsanalyses. Eerst wordt daartoe de relatie tussen locationele sorteermodellen en hedonische prijzen besproken. Locationele sorteermodellen zijn oorspronkelijk ontwikkeld als uitbreiding op hedonische prijsmodellen. Waar hedonische prijzen primair geschikt zijn voor het schatten van de bereidheid te betalen (*Willingness to Pay*; WTP) voor een marginale verandering in het lokale voorzieningenniveau, definiëren locationele sorteermodellen een WTP op basis van *endogene* grondprijzen die ontstaan in het nieuwe marktevenwicht in reactie op *niet-marginale* veranderingen in de voorzieningenniveaus. In de context van dit proefschrift zijn locationele sorteermodellen met name interessant, omdat zij een consistente interpretatie van verschillende welvaartseffecten van kapitalisering bieden. In paragraaf 7.3 worden de netwerkexternaliteiten uit hoofdstuk 4 opnieuw geïntroduceerd in het model van hoofdstuk 6; nu als *agglomeratie-externaliteiten*. Het voornaamste resultaat van deze welvaartsanalyse is dat een externaliteit ook opgevat kan worden als *endogene voorziening*, hoewel een internalisering van de waarde van deze externaliteiten in beginsel mogelijk is—waarmee de Pareto-efficiëntie van de allocatie van land hersteld zou kunnen worden. Tenslotte wordt in hoofdstuk 7 getoond hoe het model uit hoofdstuk 6 in principe ook kan worden toegepast voor het bepalen van de waarde van *open ruimte*. In de bestaande locationele sorteermodellen wordt het totale aanbod van land voor woningbouw meestal verondersteld gegeven te zijn. Omdat het sociale welvaartsniveau in ruimtelijk evenwicht in het model van hoofdstuk 6 ook afhankelijk is van het totale aantal ontwikkelde locaties, kan de waarde van open ruimte worden gerelateerd aan de bereidheid van een maatschappij af te zien van meer woonruimte in een algemeen evenwichtsbeschouwing (*general equilibrium willingness to accept*; GE-WTA). Bovendien wordt aangetoond dat met een vertaling van deze GE-WTA naar overeenkomstige opportuniteitskosten (*opportunity costs*), de betreffende locatie niet ontwikkeld zou moeten worden. Deze interpretatie biedt een theoretische basis voor het samenvoegen van de virtuele waarde van land uit waarderingsmethoden met een echte marktwaarde voor hetzelfde gebied.

Het model dat is ontwikkeld in hoofdstuk 6 wordt in hoofdstuk 8 vertaald naar een speciaal type multi-agentsysteem (*multi-agent system*). In hoofdstuk 8 wordt het geïntroduceerd als een multi-agent discreet keuzemodel (*Multi-Agent Discrete Choice Model*; MADCM). Er bestaan zowel economisch-theoretische als numeriek-theoretische redenen voor deze vertaling. In een MADCM worden individuele agenten uitgerust met een beslisregel en krijgen zij de waarden van trekkingen uit een verdeling van de persoonlijke component in de preferentiestructuur van het overeenkomstige logit-keuzemodel. Deze procedure vertoont overeenkomsten met Monte Carlo-integratiemethoden in de econometrie. Dit maakt het mogelijk het resultaat van een simulatie op te vatten als een van de vele mogelijke realisaties, in plaats van de meer abstracte duiding in termen van kansen, zoals in hoofdstuk 6. Daarnaast biedt het een consistente interpretatie van de resultaten met een eindig aantal agenten. Het hoofdstuk toont de mogelijkheid meer complexe vormen van externaliteiten op te nemen en laat zien hoe deze kunnen worden ondergebracht in een evolutionaire ontwikkeling in het model. Hoewel complexe externe effecten kunnen resulteren in patronen die er op het eerste gezicht realistisch uitzien, neemt ook het aantal mogelijke evenwichten toe. Als gevolg van het laatste kunnen alleen meer gestileerde situaties en fenomenen worden verklaard. Desalniettemin bieden deze situaties inzicht in fenomenen die belangrijk kunnen zijn voor het grondbeleid. Nadat is getoond hoe in een MADCM enkele klassieke grondgebruikmodellen kunnen worden ondergebracht, worden enkele experimenten uitgevoerd. De eerste heeft betrekking op de mogelijkheid dat het huidige grondgebruikpatroon een lock-insituatie weerspiegelt, in plaats van een optimale configuratie. Met een combinatie van evolutionaire ontwikkeling, een endogene agglomeratie—gebaseerd op sociale interacties—en de aanwezigheid van obstakels, kunnen meervoudige evenwichten worden gegroepeerd volgens verschillende typen. Er wordt een voorbeeld gepresenteerd met twee typen, waarbij de één Pareto-superieur is aan de ander. In het tweede experiment wordt het aantal locaties waaruit de agent kan kiezen om zijn welbevinden te verbeteren, beperkt. Dit kan worden beschouwd als een voorbeeld van onvolledige informatie. Simulatieruns voor specifieke parameterwaarden tonen aan dat de evolutie naar een optimaal grondgebruik enkele tijdelijke configuraties kan bevatten. Dit experiment openbaart een andere reden om voorzichtig te zijn met het opvatten van de huidige grondgebruikpatroon als zijnde optimaal. Het derde experiment heeft betrekking op een uitbreiding van de behandeling van open ruimte in hoofdstuk 7. Indien open ruimte niet alleen wordt opgevat als een puur collectief goed, maar ook als een bron van positieve externaliteiten, kan dat gevolgen hebben voor rechtvaardigheidsoverwegingen in het grondbeleid. In een model waarin de groepen met hogere inkomens zich het dichtst bij een exogeen gegeven centrum gevestigd hebben, zijn dezelfde groepen waarschijnlijk ook in staat hogere marktprijzen te betalen voor locaties grenzend aan een gebied met open ruimte, indien open ruimte wordt opgenomen als extern effect waarvan de waarde wordt gekapitaliseerd in de marktprijs voor grond. In een getoonde simulatierun wordt als gevolg hiervan de groep met het op één na hoogste inkomen weggedreven van haar oorspronkelijke locaties, waardoor deze groep wordt geconfronteerd met een groter verlies in welvaart indien externe effecten van open ruimte worden meegenomen in de analyse.

Hoofdstuk 9 bevat een samenvatting en de conclusies, die zijn gebaseerd op het beantwoorden van de onderzoeksvragen. De voornaamste conclusie luidt dat in een welvaartseconomische benadering van ruimtelijke ordening een consistente toepassing van het begrip *maatschappelijke welvaart in ruimtelijk evenwicht* belangrijker is dan het streven naar marktefficiëntie. De rol van theorieën rond complexe systemen komt naar voren in de integratie van marktinteracties met niet-marktinteracties in een economische analyse. Niet-marktinteracties kunnen hierbij worden opgevat als *endogene voorzieningen* in plaats van externe effecten. Dit perspectief leidt er toe dat de taakverdeling tussen overheid en markt minder scherp is dan in het neoklassieke kader. Verder onderzoek naar de invulling van deze taakverdeling in specifieke beleidstoepassingen is daarom noodzakelijk.

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Wilbert Grevers (1972) worked during his doctoral research as a researcher at the University of Twente. This thesis was completed at the Centre for Clean Technology and Environmental Policy. Wilbert received his Master's degree in Mechanical Engineering, with specialisation in Fluid Dynamics, from the University of Twente in 1997. Between 1997 and 2003 he worked as a consultant and modeller in different parts of the Netherlands. His main research interests are spatial economics, environmental economics, and energy economics. In the models he develops, Wilbert frequently aims at combining traditional theories with modern computational methods. He currently works as a postdoctoral researcher at the University of Groningen.